Transition Evolution and Change

Robin Matthews

Professor Emeritus
Kingston University Business School London

JM Keynes Professor of Management
Moscow School of Social and Economic Sciences
President International League of Strategic Management Assessment and Accounting
www.rustrategy.ru

+44(0)20 547 7443 (Office)
+7 095 434 3600 (0ffice)
+447766 592 904 (mob)
http://business.kingston.ac.uk/CIBP.php

Draft Please do not quote without permission of the author

TRANSITION EVOLUTION AND CHANGE

1. INTRODUCTION[1]

The paper sets out a general theory of organizational transition: that is evolution and change through forming and reforming coalitions and coalition structures on an organization matrix. Three fundamental concepts underlie the discussion; the organization matrix, the transition frontier, which together with the third concept, organizational grammar, represents a situation of self ordered criticality[2].

The organization matrix is a square matrix of activities or processes of two kinds; independent activities or stand alone activities[3] (diagonal elements of the matrix) and interdependent activities (off diagonal elements) that are combinations of activities, joint products (externalities, synergies or complementarities the impact of one activity upon another). The organization matrix is an extremely general concept, applicable to many types of organization, businesses, institutions, public private or voluntary and to entire societies. We should think of the organization matrix as a hierarchy of matrices, since organizations as such are composed of coalitions within coalitions, integrated hierarchies of activities, beginning, for example with fundamental activities or teams, leading to coalitions of the teams that make up the firm's projects and then to coalitions of projects that form business units. Corporations are coalitions of business units or divisions that may form themselves into coalitions, in the form of supply chains, joint ventures, partnerships and alliances. Treatment of interdependence activities follows Arrow's (1969) treatment of externalities as separate goods or services.

The process of forming larger and more complex coalitions as we ascend the hierarchy of the organization matrix opens up of new possibilities: new synergies from the coalitions that emerge that are the basis of evolution at different levels of hierarchy of the organization matrix. Thus organization matrix theory it encompasses industry or sectoral evolution[4] and firm based theories which parallels discussions about the level at which evolution takes place in biological systems[5]

Transition refers here simply to formation and reformation of coalitions into coalition structures and embodies both evolution and change. Evolution is a subset of change

[1] References below are indicative only. A full set of references is available on request.
[2] Jensen (1998).
[3] For a definition of activities see Georgescu Roegen (1966, 1969) Koopmans (1960) and below.
[4] for stress upon industry changes, rates of entry see Hannan and Freeman (1983,) HAnnan and Carrol (1992, 1995) generally structure conduct and performance models (see for example Porter, (1980). For firm based approaches see Mcpherson (1983) and generally resource based approaches, for example, Wernerfeldt (1984). For focus on eveolution at the level of routines (our organizational grammar) see Nelson and Winter (1982) and for example, Teece Pisano and Shuen (1997).

[5] See Gould (2002).

implying an increase in fitness. The meaning of fitness in this context is defined as the extent to which they achieve their objectives, or payoffs. Payoffs have the same role as utilities or transferable utilities in economics or fitness values in evolutionary theory: they put a value on states, in this case activities (or coalitions of activities) and hence implicitly define goals. Thinking in terms of payoffs means that evolutionary change is not tied to a single motivation, the search for higher profits but is broadly defined to encompass a variety of stakeholder interests. Thus the theory based upon the organization matrix can be the foundation for a stakeholder approach to organizations.

A coalition structure is defined by the set or configuration of coalitions it contains[6]. Figure 1 (below), the transition frontier, maps coalition structures distinguished by the largest coalition (the largest number of fundamental activities) contained in the structure, against the probability a particular structure occurring, on the assumption that the partition of activities into coalitions is random: so if there are n activities, there are 2^n coalitions and the probability of an activity being a member of a particular coalition is 2^{-n}. The transition frontier is thus based upon Maxwell – Boltzman statistics. At the macro level, it is an attractor, self ordered in the sense that coalitions gravitate to it, simply as a result of the binary choice (join a coalition or not). Coalitions and coalition structures are continually in flux, so the transition frontier is a state of macro equilibrium only in so far as it is probability distribution of coalition structures. Adding the third concept into the picture the transition frontier illustrates a situation of self ordered criticality: self ordered in the sense that organizations, as coalitions of activities gravitate to it without external tuning and critical in the sense that it they are attracted to a point on which change on all scales is possible. Further, as explained in the paper, the transition frontier may express a kind of power law, underlying organizational transition that is summarized by the familiar formula of combinatorial calculus,

The transition frontier is drawn on the assumption that things are random or ergodic, either in the sense that coalitions occur randomly, which might be a proposition arising from the viewpoint of classical statistical mechanics, or in the sense that choice itself is random, which might be a proposition arising from a spin glass approach. For any number of reason things are not likely to be so. An alternative might be Kauffman's NK approach, with N being the number of fundamental activities (diagonal elements in the matrix) and K the degree of connectivity (the number of non zero off diagonal elements)[7]. The organization matrix in Kauffman terms might be seen as a partitioned landscape offering limited possibilities for coalition formation. None of these three alternatives is offered in the view of change that follows. Instead the route is through organizational grammar.

Organizational grammar describes the rules, laws, treaties, agreements, cultures, traditions and conventions that together with the attitudes, values, motivations and mind sets that exist in society in general and in the people associated with the organization. Organizational grammar has many dimensions; external and internal, formal and informal, social and personal. We can

---

[6] Aumann and Hart (1994), Myerson (1991).

[7] Kauffman (1993, 1996, 2001) for NK systems. On the statistical Mechanics and Spin Glass approaches see Sklar (1993), Durlauf (1996), Anderson Arrow and Pines (1993). For a discussion of organizational grammar see Matthews (1005, 2006, and 2008 forthcoming).

think of two general features of grammar; morphology and syntax. As a first approximation, activities, or the way we choose to define activities, constitute the morphology of organizational grammar and syntax corresponds to the connections that exist between the nodes when coalitions are formed on the organization matrix. Organizational grammar performs at least 3 functions: first, it determines coalition formation and the extent to which coalitions are glued together; second grammar is designed to limit change to manageable or bearable proportions; third grammar is the source of information, communication and meaning.

In this third respect, organizational grammar as language, the idea is founded on Wittgenstein's conception of the role of grammar in language games, as "rules for the use of a word" that "determine meaning" and his persistent analogy that "grammar … somewhat the same relation to language as … the rules of the game have to the game".[8] Organizational grammar, in the paper, extends the idea of grammar to organizations and to the rules, conventions and mind sets that govern coalition formation on the organization matrix. The concept of also draws on extensions of the idea of language games by Francois Lyotard and Foucault's notions of the archaeology and genealogy that governs attitudes and policies historically and perhaps most important, determines the discourse that underlies the interpretation of business and society.

The impact of organizational grammar means that the process of coalition formation and reformation on the organization matrix as represented on the transition frontier, self organizes into a critical state; a state of self ordered criticality where change on all scales is possible. In so far as organizational grammar is successful in limiting change in the inner dynamics of organizations, it allows pressures in the organizations environment, the outer dynamics to build up: pressures that may on the one hand percolate throughout the hierarchy of the organizations, giving rise to dramatic change, revolutions in the social or organizational worlds, akin to avalanches or earthquakes in the physical worlds. On the other hand change may be limited; organizational grammar may prevent change from percolating throughout the matrix.

The process of forming larger and more complex coalitions as we ascend the hierarchy of the organization matrix opens up of new possibilities. Thus evolution can take place at many levels. Many activities are involved in transition. The system state is defined (at any point in time) as follows; by the inner dynamics, the configuration of activities into coalitions; by the outer dynamics, competition, technology, demographics macroeconomics and so on; by the payoffs generated by organizations; and finally by organizational grammar itself. All four elements of the system states are complex adaptive systems: so the roots of the theory are in complexity[9].

Returning to the first function, the extent to which coalitions are glued together, clearly the theory draws on co-operative games. In summary the theory which we describe subsequently, is a complex adaptive system, with n activities and m possible states. In this paper we focus on binary choice: in other papers, possibilities of higher degrees of choice and the relationship with coalition games have been more closely examined.

---

[8] Wittgenstein (1963, 1987).
[9] Coveney and Highfield (1995), Mainzer (2004).

Change and demand are at the root of the theory presented here. Equilibrium in the theory described in this paper is a probability distribution over a set of points and not a single point. Coalitions at the micro level are in a state of flux; change occurs, within coalitions and coalition structures change. Coalitions at every level of the organization matrix are in a sense consumers of activities (and coalitions) at a lower level. High level coalition structure reflect payoffs associated with final demand. Thus the approach is Keynesian, orientated on demand, in contrast to the supply driven models of modern growth theory.

Further the emphasis is upon distribution rather than pricing. In contrast to a neoclassical approach in which distribution is more or less a subset of the pricing process, in organization matrix, prices follows the distribution process.

The paper develops in the following way.

Section 2 describes the mapping of the organization matrix as a set of interdependent activities into coalitions and coalitions structures via a payoff matrix. Feasibility conditions are set out.

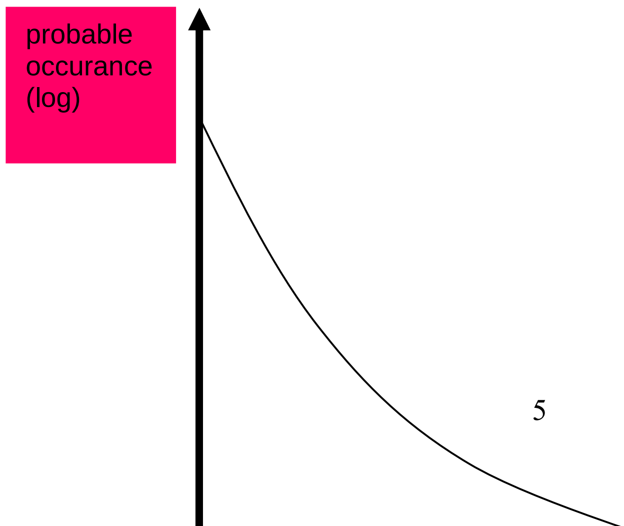
Section 3 analyses payoffs. The organization matrix satisfies conditions for the existence of Shapley values. Under reasonable conditions, actual payoffs are shown to be weighted averages of Shapley values.

In Section 4 the process of coalition formation on the organization matrix is described, introducing two notions; the search matrix and the choice matrix. Equilibrium on the organization matrix is a probability distribution: motivations for changing coalitions underlie the disequilibrium nature of the underlying microstates.

Section 5 sets out a simplified example to illustrate the previous sections.

Varieties of search are described in Section 6, contrasting an approach through organizational grammar with other approaches in the literature.

Section 7 contains some concluding observations.



2. THE ORGANIZATION MATRIX

The organization matrix may be written as a matrix of activities or as a matrix of payoffs. As a square (n x n) matrix of activities it is

$$\boldsymbol{A} = [Ajk] \qquad (1.1).$$

Here ***A*** denotes the level at which the n activities are utilised. The organization matrix can also be written as a square (n x n) matrix ***a***

$$\boldsymbol{a} = [ajk] \qquad (1.1a).$$

The matrix ***a*** is a list of assets underlying the activities. Activities [Ajk] are the actions carried out by assets; the flows of services made possible by assets. Activities describe physical operations; manufacturing, producing, storing, marketing, selling, throwing something away or consuming something in a particular way that embodies accumulated knowledge and experience. Activities are the building blocks of organizations.

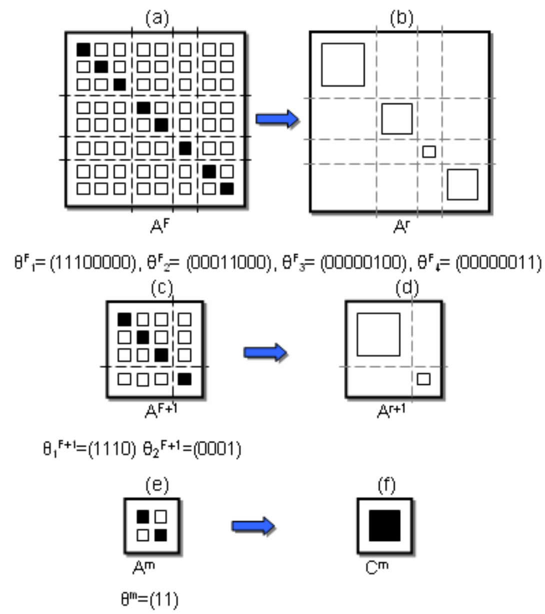


Figure 2

Diagonal elements (*j = k)* in the organization matrix [Ajk] are independent activities and off diagonal elements (*j ≠ k)* interdependent activities. In Figure 1(a), the shaded diagonal boxes illustrate independent activities and non shaded off diagonal boxes, joint activities, in a simplified (8 x 8, n = 8) organization matrix. Coalitions in 1(a) are formed to capture the payoffs from interdependent activities. In Figure 1(b), a coalition is formed of the first three activities, denoted by the operator[10] θ_1^F and the string (11100000) which are combined to form coalition one, the upper left hand box in Figure 1(b). Similarly coalitions two (two member), three (one member) and four (two member), are denoted θ_2^F = (00011000), θ_3^F = (00000100) and θ_4^F = (00000011). The process of organizational evolution takes place through forming and reforming on an organization matrix of made up of large, and perhaps increasing numbers of potential activities, in response to changing conditions on the organization matrix.

As illustrated in Figure 2, organizations are hierarchical structures of activities, linked by agents, who form successive coalitions. Activities, linked in coalitions at one level of the organization matrix, become diagonal elements at a higher level. New coalition possibilities may then appear, illustrating emergent properties of the organization matrix, as we proceed up the hierarchy. The organization matrix can be thought of as formed from fundamental activities $\boldsymbol{A^F}$, existing at a basic level in the hierarchy that are combined potential coalitions $\boldsymbol{A^R}$ and become diagonal elements at higher levels of the hierarchy $\boldsymbol{A^{F+1}}$, where new coalition possibilities, may occur and are represented in turn as $\boldsymbol{A^{R+1}}$, off diagonal elements at still higher levels, $\boldsymbol{A^{F+2}}$ and so on. The process of diagonalisation and further combination continues up the hierarchy. Thus the organization

[10] The interpretation of the operator θ is set out below.

matrix becomes a set of complex coalitions of increasingly larger numbers of fundamental activities: coalitions within coalitions, representing longer and longer chains or networks of linked activities.

If Figure 1 is seen as a process of forming coalitions at successively higher levels of the organization matrix, Figure 2 illustrates the process of forming fundamental activities $\boldsymbol{A^F}$ in figure 1(a) into 4 possible (C^1, …., C^4) corresponding to $\boldsymbol{A^r}$ in 1(b). New higher level higher level complementarities may appear, ($\boldsymbol{A^{F+1}}$, in 1(c)), and be combined into two possible coalitions $\boldsymbol{A^{r+1}}$, which in turn form a higher level organization matrix $\boldsymbol{A^m}$. The simplified situation illustrated by the two Figures, results in the formation of a grand coalition ($\boldsymbol{C^m}$). From the fundamental activities $\boldsymbol{A^F}$**,** a series of coalition structures are formed, $L^P{}_F$, $L^P{}_{F+1}$, $L^P{}_{F+2}$, $L^P{}_{F+m}$ in Figure 2, corresponding to Figures 1(a) through 1(f) and decisions (illustrated by the binary strings) θ^F, θ^{F+1} ,θ^m . A coalition structure, defined more formally below, describes the set of coalitions currently existing on the organization matrix. Coalitions and coalition structures consist of coalitions within coalitions, generally consisting of increasing numbers of fundamental activities as we proceed up the hierarchical organization matrix.

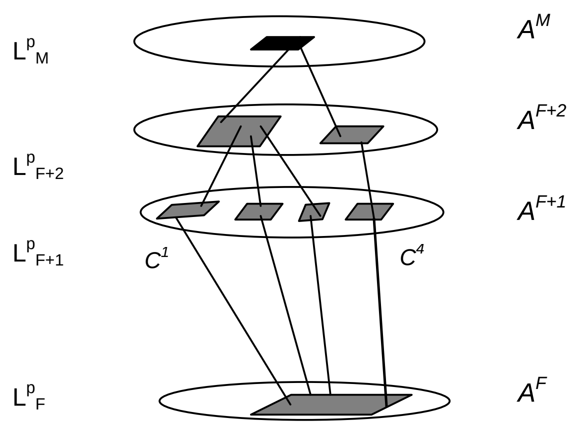


Figure 2

Alternatively the organization matrix illustrated in the two Figures can be written in terms of payoffs, as an (n x n) matrix $\boldsymbol{B}$ of payoffs, generated by activities.

$$\boldsymbol{B} = [Bjk] \qquad (1.2)$$

Bjk corresponding to activities, *Ajk* are created at every level of the organization matrix. A hierarchy of payoffs corresponding to the hierarchy of activities can be defined starting with payoffs from fundamental activities, written as $\boldsymbol{B^F}$.

Where $j = k$ we have payoffs from activities as stand alone entities and where $j \neq k$ we have payoffs from joint activities or contributions of activity j to activity k as seen by agents in k. Payoffs are represented in an n x n matrix ***B*** (***B*** = [*Bjk*]. Payoffs (which may be positive or negative), can be conceived of in a number of ways; as a surplus (deficit) of benefits over costs in monetary terms, as outcomes, tangible or intangible, as fitness values in organizations' struggle for survival, or as transferable utilities. Defined as fitness values, higher payoffs do not necessarily imply better or worse situation but merely a different set.

Coalition formation

Organizational evolution, at every level, reflects an attempt by agents to internalize payoffs through coalition formation and reformation. A coalition consisting of a union of K activities (k = 1,2, …., K; K≤ n) is written as $\boldsymbol{C^k}$;

$$\bigcup_{j,k=1}^{K} Ajk = C^k \qquad (1.3)$$

or it can be written as a list of K assets, c^k

$$\bigcup_{j,k=1}^{K} ajk = c^k \qquad (1.3a)$$

Payoffs from coalition $\boldsymbol{C^k}$, are denoted $\boldsymbol{B^k}$, a real number (not necessarily positive). Every coalition $\boldsymbol{C^k}$ has corresponding (potential) payoffs $\boldsymbol{B^k}$ and

$$\boldsymbol{B^k} = \sum_{j} \sum_{\substack{k \\ j,k \in C^k}} Bjk \qquad (1.4).$$

When K = 1, we have a single member (activity) coalition $Bjk = \boldsymbol{B^k}$, and when K > 1 we have coalitions made up of groups of activities.

The transformation of activities, ***A***, into payoffs, ***B***, is achieved by a mapping ***W*** of ***A***, (or a mapping of a coalition $\boldsymbol{C^k}$ into the real number system, where, ***W*** is diagonal matrix such that

$$\boldsymbol{W} = \text{diag}(w_1, w_2, \ldots\ldots, wk) \qquad (1.5)$$

And

$$\boldsymbol{B^k} = \boldsymbol{WC^k} \qquad (1.6)$$

So that

$$\boldsymbol{B^k} = \sum_{j=1}^{K} \sum_{k=1}^{K} wjAjk \qquad (j,\ k = 1,2,\ldots\ldots.n) \qquad (1.6a)$$

$\boldsymbol{B^k}$ denotes a (K x K) coalition (of K interdependent activities) and *[Bjk]= [wjAjk]* (j, k = 1,2,…….K; K ≤ n), an associated matrix of coalition payoffs, generated by activity k. Where j = k we have the stand alone payoff to an activity j: where j ≠ k we have an external effect which may be positive or negative.

W may be interpreted provisionally as marginal value (or marginal revenue) products, ($w_j = \dfrac{\partial\ B^k}{\partial A_J}$, j, k = 1, 2, …..K; $A_j \subseteq C^k$)[11], attaching values *wj,* to incremental increases in activity j to each of the K activities *[Ajk]* in coalition $\boldsymbol{C^k}$. We show later that actual payoffs $\boldsymbol{f_k}$ may differ from the value of the marginal contribution ($w_j A_{jk}$) of activity j to a coalition.

Feasibility

Large numbers of possible partitions of the organization matrix exist and not every partition may be feasible. Hence we must set out the feasibility conditions for a coalition. Conditions change. A set of payoffs, realised or anticipated, that satisfy feasibility conditions at one time, may fail to do so at another; giving sources of emergence on the organization matrix, in addition to the emergence of new coalition possibilities in the hierarchy of the organization matrix.

The feasibility conditions we set out for coalitions are straightforward; (i) the payoff value of a coalition as a whole should be positive, (ii) the payoffs, f_k, distributed to every activity, k, in the coalition, $\boldsymbol{C^k}$, should not be negative[12] (iii) the sum the payoffs to individual members of the coalition should not exceed payoffs to the coalition as a whole;

i. $\boldsymbol{B^k} > 0$

ii. $f_k \geq 0$ all (k = 1,2,……K) and

iii. $\sum_{k=1}^{K} fk \leq B^k$ (1.7)

The organization matrix satisfies conditions for the existence of a set of payoffs equivalent to Shapley values for a coalition and that given the assumption that possible distributions form a convex set, feasible payoffs can be expressed as weighted Shapley values.

Coalition structures

---

[11] In full equilibrium and assuming free disposability, we can think of activity levels being set at levels that equate wj in all its uses, joint as well as stand alone activities. In outlining an evolutionary model we are concerned with disequilibrium situations and we explain below, how ***W*** may be drawn from a wider set of values.

[12] Generally $\sum_k f_k = B^k f_k$ but there is no reason to suppose that $f_k = \sum_k w_k Akj (= \sum_k Bkj)$ but as discussed below they are identified with a wider set of weighted Shapley values and not confined to marginal value or marginal revenue products.

We can illustrate the distinction between coalitions and coalition structures with reference to the Figures above. In 1(b) and 1(c) for example, the four coalitions make up a possible coalition structure, illustrated in Figure 2 (L^{P}_{F+1}). Similarly Figures 1(d) and 1(e) illustrate a coalition structure consisting of two coalitions (L^{P}_{F+2}) in Figure 2.

More formally, a coalition structure ***L*** is a partition of activities on ***A*** and is described by the set of R coalitions,

$$\boldsymbol{L} = \{\boldsymbol{C^1}, \boldsymbol{C^2},\ldots,\boldsymbol{C^k},\ldots, \boldsymbol{C^R}\} \qquad (1.16).$$

Or in terms of payoffs

$$\boldsymbol{L} = \{ \boldsymbol{B^2}, \boldsymbol{B^2},\ldots \boldsymbol{B^k},\ldots, \boldsymbol{B^R}\} \qquad (1.16a).$$

In 1.14 and 1.14(a), *L* is a partition of the organization matrix ***A*** (an n x n matrix) into a coalition structure, made up of R separate coalitions each made up of K activities, (such that $0 \leq K \leq n$ and $\sum_{K \varepsilon L} K = n$) that satisfies the following conditions;

i. $\boldsymbol{B^r} \geq 0$ all $B^r \in \boldsymbol{L}$
ii. $\boldsymbol{B^k} \cap \boldsymbol{B^j} = \varnothing$, all $k \neq j \in \boldsymbol{L}$
iii. $\bigcup_{C^r \in L} \boldsymbol{C^r} = R \quad (R \leq n) \qquad (1.17)$

Identification of coalition structures is an NP hard problem[13]: the number of structures increases exponentially with n, the number of fundamental activities on the organization matrix. Let $\boldsymbol{L^{\Pi}}$ be the number of possible coalition structures and let $\boldsymbol{L^{p}}$ be the set of feasible coalitions and coalition structures identified by agents. For large numbers of fundamental activities

$$\boldsymbol{L^{p}} \subseteq \boldsymbol{L^{\Pi}} \qquad (1.18)$$

Table 1 sketches the set of coalitions and coalition structures for n = 4 fundamental activities. As demonstrated below[14], the two coalitions in Figure 1(e) that comprise L^{P}_{F+2} in Figure 2, represent just 1 of the 52 possible coalition structures ($\boldsymbol{L^{\Pi}}$=52) that can be formed out of 4 coalitions (Figure 1(c)).

---

[13] Algorithms used to describe computable problems are of two classes, based on the length of time it takes to find a solution to a problem as a function of some number N that measured its size. Polynomial problems (algebraic power of N, N^2, or N^3 and so on) that are said to be tractable are such that the length of time required to crack them does not become unbounded as the size increases: class P. NP problems are such that the length of time increases in an exponential fashion (to the power N) are intractable because the length of time required to solve them spirals out of control (Coveney and Highfield, 1995).

[14] See Table 2 below.

3. DISTRIBUTION

In this section we show that payoffs satisfying feasibility conditions (1.7) exist that can be expressed as weighted Shapley values. The properties of the organization matrix ensure that Shapley values exist. If we assume that the set of possible payoffs is a convex set, then weighted Shapley values exist. Realised payoffs which may differ from anticipated payoffs $\boldsymbol{f_k}$ can be expressed as weighted Shapley values. We also relate the elements of $\boldsymbol{W}$ to weighted Shapley values and therefore to $\boldsymbol{f_k}$.

We begin by distinguishing four payoff concepts;

a) Marginal contribution of activity to a coalition
b) Value added by an activity to a coalition.
c) Shapley value of an activity to a coalition
d) Weighted Shapley value of an activity to a coalition.

a) The marginal contribution of an activity to a coalition mc_j

$\boldsymbol{B^k}$ represents the payoff values of a coalition $\boldsymbol{C^k}$ made up of K activities. The contribution of an activity j, mc_j to coalition $\boldsymbol{C^k}$ is defined as;

$$mc_j = \sum_{k=1}^{K} Bjk \quad (k = 1, 2, ..., K)$$
$$= wj \sum_{k=1}^{K} Ajk \quad (A_{jk,} B_{jk} \subseteq C^k) \qquad (1.8)$$

The marginal contribution of an activity is its row value in a coalition; that is the marginal value (or marginal revenue) product of activity j to the coalition as a whole. The payoff value of coalition $\boldsymbol{B^k}$ is the sum of the values of the j (j ε $\boldsymbol{C^k}$). Even though marginal contributions to that coalition satisfy the feasibility conditions in (1.7)[15] incentives to change coalitions or coalition structures may exist.

b) The value added by activities to a coalition, mv_j

In addition to the marginal contribution of an activity j to a coalition, (that is, Bjk, the outward synergies or complementarities from j to k), it may be that activities k have a reciprocal impact on j (that is Bkj; the inward synergies or complementarities to j from k). Both effects influence the value added by activity j to a coalition which is the difference between the value of that coalition with ($\boldsymbol{B^k}$) and without j ($\boldsymbol{B^{k/j}}$); that is[16]

$$mv_j = \left[B^k - B^{k/j} \right] \qquad (1.9),$$

[15] Free disposability is also required for (1.7ii).

[16] Sometimes written as $\left[[v(C^k \cup \{s\}) - v(C^k)\right]$,v(.) being the value of coalition $\boldsymbol{C^k}$ with and without activity s.

or,

$$mv_j = \sum_{allk \in C^k}^{K} Bkj + \sum_{\substack{allj \in C^k \\ k \neq j}}^{K} Bjk \qquad (1.9a)$$

c) Shapley values as payoffs

The Shapley value φ_j of an activity in a coalition $\boldsymbol{C^k}$ of K, is such that sum of the Shapley values of payoffs of all activities equals the total payoffs to that coalition.

$$\sum_{j \subseteq C^k} \varphi_j = \boldsymbol{B^k} \qquad (1.10)$$

Given a coalition $\boldsymbol{C^k}$ of K activities, the Shapley value φ_j of every activity in the coalition is the weighted average of the marginal values of that activity to all sub coalitions of $\boldsymbol{C^k}$ which it is a member. The weights are the number of ways a coalition containing k can occur as a proportion of the 2^K possible sub coalitions of $\boldsymbol{C^k}$.

The Shapley value of any activity s in a coalition can be written

$$\varphi_s = \sum_{s \subseteq K-s} \frac{s!(K-s-1)!}{K!} \left[B^k - B^{k/s} \right] \qquad (1.11)$$

Or

$$\varphi_s = \sum_{S \subseteq K-k} \frac{s!(K-s-1)!}{K!} mv_s \qquad (1.11a)$$

In the expression, K is the size of coalition $\boldsymbol{C^k}$. There are K! permutation of the K agents in the coalition. Consider the set s of sub coalitions of $\boldsymbol{C^k}$, not containing activity k, ($s \subseteq K-k$). There are exactly s! ways in which other members of $\boldsymbol{C^k}$ can form a coalition without k and exactly (K - s - 1)! ways in which the remaining (K - s -1) can enter $\boldsymbol{C^k}$ after them. Expressed as a proportion of the total number of permutation of activities in $\boldsymbol{C^k}$, K!, we have the probability of a particular coalition containing activity K occurring. The final term on the right hand side of the equation is the marginal value of activity s ($mv_s = \left[B^k - B^{k/s} \right]$ to any sub coalition of $\boldsymbol{C^k}$. Hence the Shapley value φ_s is arrived at by multiplying all the possible marginal values of activity k the sub coalitions of $\boldsymbol{C^k}$ by the probability each sub coalition.

Given (1.10) if Shapley values in a coalition $\boldsymbol{C^k}$ are positive they satisfy the feasibility conditions for coalitions set out in (1.7).

d) Weighted Shapley values as feasible distributions

Actual payoffs to stakeholders in a coalition C^k may result of a separate bargaining process (or power struggle) within the coalition coinciding neither with their value added nor their marginal contribution. Hence it is necessary to demonstrate that a wide set of possible distributions exist. We also have to show that a mapping from activities into payoffs (such as ***W*** above) also exists. A weighted average of Shapley distributions satisfies this requirement but for such a range of distributions to exist, an additional property is required of the organization matrix; that the set of feasible distributions of payoffs to any activity or agent s, $\boldsymbol{F^S}$ is a convex set; that is that a set μ_s exists such that

$$\sum_{all\, s \subseteq C^k} \mu_s f_s \subseteq F^s$$

$$\text{For } 0 \le \mu_s \le 1 \text{ and } \sum_{all\, s \subseteq C^k} \mu_s = 1 \qquad (1.12).$$

Since we will rely upon weighted Shapley values we must show that (i) the properties of the organization matrix are such that Shapley values exist and (ii) that Shapley values, and in particular weighted Shapley values constitute a possible set of payoffs.
For the latter property to hold we need the additional assumption that possible payoff distributions form a convex set as defined by (1.12).

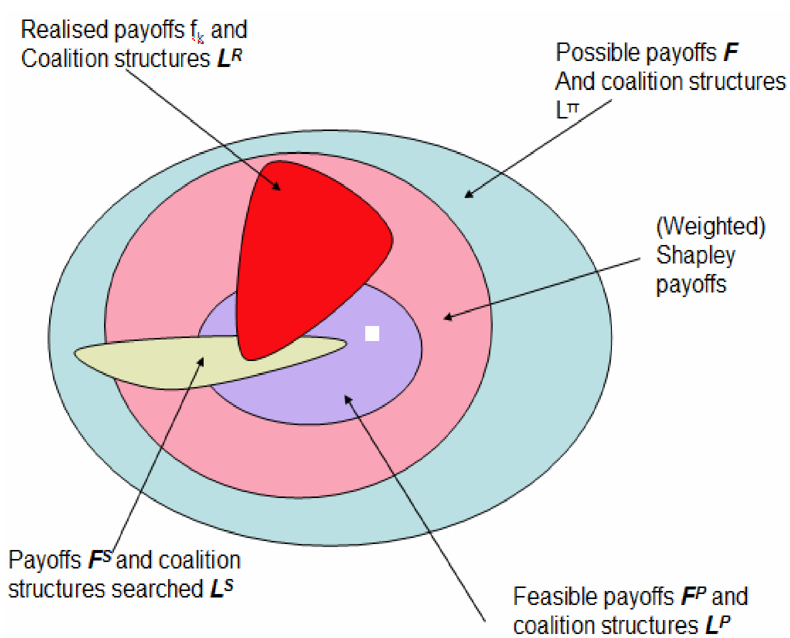


Figure 3

We define a set of weighted Shapley values, λ_s, such that .

$$\sum_{s \subseteq C^k} \lambda_s \varphi_s = \boldsymbol{B^k} \qquad (1.13)$$

where

$$i.\ \lambda_s = \sigma_s \frac{B^k}{\varphi_s}$$

$$and \qquad (1.14)$$

$$ii. 0 \leq \sigma_s \leq 1 \ and \ \sum_s \sigma_s = 1$$

Fundamental equation for realised payoffs

First consider marginal contributions to a coalition, if they are positive they are also feasible. Setting

$$\sigma_s{*} = \frac{mc_s}{B^k}$$

$$with\; 0 \le \sigma_s{*} \le 1 \;and\; \sum_s \sigma_s{*} = 1.$$

$$\sigma_s{**} = \frac{f_s}{B^k}$$

So

$$\lambda_s{*} = \sigma_s{*}\frac{B^k}{\varphi_s}\; and \sum_s \lambda_s{*} = \sum_k \sigma_s{*}\frac{B^k}{\varphi_s}$$

so marginal contributions can be defined as weighted Shapley values; that is

$$\sum_s \lambda_s{*}\varphi_s = \sum_s mc_s$$

By defining σ_s appropriately, for example $\sigma_s{**} = \frac{f_s}{B^k}$, we can see that any feasible payoff to s can be expressed as a weighted average of Shapley values.

Recalling that $mc_s = w_s \sum_{sj} A_{sj}$, we can express the mapping $\boldsymbol{W}$ in terms of Shapley values; that is [17]

$$w_s = \lambda_s^f \frac{\varphi_k}{\sum_k A_{kj}} \tag{1.14a}$$

In general,

$$\boldsymbol{W = \Psi\Lambda X} \tag{1.15}$$

where Ψ is a diagonal matrix of Shapley values, Λ is a diagonal matrix of Shapley weights[18] as defined above and Ψ, Λ and X are square (KxK) matrices such that

$\boldsymbol{\Psi}$ = Diag ($\varphi_1, \varphi_2, ..., \varphi_K$),
$\boldsymbol{\Lambda}$ = Diag ($\lambda_1, \lambda_2, ..., \lambda_K$) and

---

[17] Equation (1.14) follows from the relationship $\lambda_s^{mc} = \frac{mc_s}{\varphi_s} = \frac{w_s \sum_s A_{sk}}{\varphi_s}$.

[18] $\Lambda = Diag(\sigma_1 \frac{B^k}{\varphi_1}, \sigma_2 \frac{B^k}{\varphi_2}, ..., \sigma_K \frac{B^k}{\varphi_K})$

$$\boldsymbol{X} = \text{Diag}\left(\frac{1}{\sum_s A_{s1}}{}_1, \frac{1}{\sum_s A_{s2}}, \ ,........, \frac{1}{\sum_s A_{sK}}{}_K\right). \qquad (1.15a)$$

So we have a distinction between the marginal value of an activity to a coalition (either in terms of Shapley values or marginal contributions) and its anticipated or actual return. This in itself is a source of disturbance, change and perhaps the emergence of novelty on the organization matrix.

4. STRATEGY: SEARCH CHOICE AND ADAPTATION

We think of the strategic decision process as divided into search of possible alternatives for action, choice (and implementation) of a particular set of alternatives and adaptation as circumstances change. Extending the distinction between possible and chosen coalition structures, we consider search to take two forms:

i. search among what is already known and
ii. search in the sense of discovery.

We refer to the first kind as search of the potential domain ***P*** and of the second kind as search of the imaginal domain, ***Π,*** containing possibilities that exist, but are yet to be discovered: in Figure 3 they are subsumed within $\boldsymbol{L^{\Pi}}$. The domain of time and space in which actual choices are realised is denoted ***R.*** As n, the number of activities increases, a enormous number of possible coalition structures come to exist. This is evident even for four activities in Table 1. Normally search, as in Figure 3 is limited, by the costs of search and by bounded rationality on the part of agents. Denote the subset of possible coalitions and coalition structures actually searched as $\boldsymbol{L^{S}}$ ($\boldsymbol{L^{S}} \subseteq \boldsymbol{L^{\Pi}}$).

Search

Evolution, through forming and reforming coalitions between activities, is a continuous process, made up of the following sub-processes:

a. Search ***S*** in the two domains ***P*** and ***Π,*** by decision making agents, looking for higher and higher levels of payoffs.
b. Choice and the decision $\boldsymbol{\theta}$ to form and implement coalitions and coalition structures and realise payoffs in ***R***, the domain of time and space.
c. Adaptation.

We confine discussion of search to the first kind: within ***P.*** Search partitions the organization matrix into a set $\boldsymbol{L^{P}}$ containing feasible coalition structures. Figure 1(b) for example illustrates a partition of the organization matrix in 1(a) into a possible structure of 4 coalitions. Choice in 1(c) represents commitment to implement particular coalitions and structures. Since evolution is through shifting coalitions and coalition structures on the organization matrix, we now consider their formation, break-up and re-formation by introducing search matrices, ***S***, which partition the organization matrix at various stages of the hierarchy (1(b),1(d) and 1(f)) and choice matrices $\boldsymbol{\Theta}$ (1(c) and 1(e) and Figure 2).

Search matrices (***S***)

We represent search for a coalition by an (n x n) diagonal search matrix $\boldsymbol{S_r}$ that partitions the organization matrix, ***A,*** in terms of activities (or ***B,*** payoffs), into a coalition $\boldsymbol{C^k}$ of size K. The organization matrix in Figure 1(b) is one possible partition via the operation $\boldsymbol{S_r}$ of 1(a). $\boldsymbol{S_r}$ is an adaptation of an n x n, identity matrix with 1's on the diagonal (corresponding to the members of coalition $\boldsymbol{C^k}$) and 0 elsewhere. The search process for a coalition is represented by a square matrix $\boldsymbol{S_r}$;

Definition

i. $\boldsymbol{S_r} = \|\delta jk\|$, (j = k and j,k = 1, 2,…. n)
ii. With δjk = 1, if and only if activities j and k are included in the potential coalition $\boldsymbol{C^k}$ and
iii. δjk = 0 otherwise. (1.19).

Let $\boldsymbol{A^F}$ be an organization matrix made up of fundamental activities. Search for a coalition in activity space is represented by the operation

$$\boldsymbol{S_r' A^F S_r} \tag{1.20}$$

and search for a coalition in payoff space by the operation

$$\boldsymbol{S_r' B^F S_r} \tag{1.20a}$$

We can express a possible partition of the (n x n) organization matrix $\boldsymbol{A^F}$ into a set of R coalitions as $\boldsymbol{A^r}$, where,

$$\sum_{r=1}^{R} \boldsymbol{S_r' A^F S_r = A^r} \tag{1.21}$$

Generally the search matrix partitions the organization matrix in to a block matrix, setting out a coalition structure ***L*** (a structure of R coalitions) in block form.

Alternatively the search operation can be expressed as payoffs

$$\sum_{r=1}^{R} \boldsymbol{S_r' B^F S_r = B^r} \tag{1.21a}$$

In (1.19) and (1.19a) R, the number of coalitions is such that $1 \leq R \leq n$ and K, total number of activities is such that $\sum K \leq n, all\, K \subseteq C^k$, $all\, C^k \subseteq L^R$, where n the number of activities.[19]

Choice

[19] As we proceed up the hierarchy illustrated in Figure 2, we see that n at that particular level can be expressed either as fundamental activities or coalitions of fundamental activities.

Equations (1.19) and (1.19a) partition the organization matrix into a possible coalition structure $\boldsymbol{L}$. As noted above selection among coalition structures is an NP hard problem. Choice of a particular coalition structure is indicated by a choice matrix $\boldsymbol{\Theta}$. $\boldsymbol{\Theta}$ is made up of a column of R row vectors, (choice vectors, $\boldsymbol{\theta_1, \theta_2, \ldots \theta_k, \ldots, \theta_R}$,) where R ($R \leq n$) is the number of coalitions in the structure. A particular choice vector $\boldsymbol{\theta_k}$ ($\boldsymbol{\theta_k} \subseteq \boldsymbol{\Theta}$) has 1 in the row if a the k^{th} activity is included in R^{th} coalition and 0 otherwise: the number of activities in a coalition is equal to number of 1's in the R^{th} row.

Thus we define the choice matrix $\boldsymbol{\Theta}$;

Choice matrix: definition

i. $\boldsymbol{\Theta}$ is an R x n matrix of 1's or 0's.
ii. R, the number of rows, is the number of coalitions in the chosen coalition structure. The total number of activities in the R^{th} coalition is measured by the number of 1's in the R^{th} row.
iii. The R^{th} row of the $\boldsymbol{\Theta}$ matrix consists of 1's and 0's with 1 in column k if the k^{th} activity is included in coalition R and 0 otherwise.

(1.22)

In Figure 1, the coalition structure in (c) is formed by operating choice vectors (11100000, 00011000, 00000100, 00000011), on the partitioned matrix in (b) ($\boldsymbol{A^r}$ or $\boldsymbol{B^r}$).

We now set out criteria for agent choice of a coalition to join and therefore of a coalition structure. Let $\boldsymbol{L^{\Pi}}$ be the set of possible coalition structures that may be objects of search and let $\boldsymbol{L^S}$ be the set of structures actually searched ($\boldsymbol{L^S} \subseteq \boldsymbol{L^{\Pi}}$), that have an array of anticipated payoffs $\boldsymbol{F^k}$. Suppose $\boldsymbol{L}$ is the chosen coalition structure ($\boldsymbol{L} \subseteq \boldsymbol{L^S}$) with anticipated payoffs to coalition members $\boldsymbol{f_k}$. To be a candidate for selection we require (i) expected payoffs to an activity in the chosen coalition should be at least as high as payoffs in an alternative coalition, $\boldsymbol{f_k} \geq \boldsymbol{F^k}$ (all $\boldsymbol{F^k} \in \boldsymbol{L^S}$). and (ii) every activity is expected to make a positive marginal contribution if they are to be accepted as a member of a particular coalition.

Choice criteria: definition

i. $f_k \geq \boldsymbol{F_k}$ (all $\boldsymbol{F_k} \in \boldsymbol{L^R}$) and
ii. $mc_k \geq 0$ *all* $mc_k \in C^k$ *and all* $C^k \in L$ (1.23)

Suppose the conditions set out in (1.21) are met by a coalition $\boldsymbol{C^k}$ one of R coalitions structure $\boldsymbol{L}$. The decision is carried out by an (n x 1) column vector $\boldsymbol{\theta_k}$. $\boldsymbol{\theta_k}$ has 1 in the k^{th} position if the k^{th} activity is included in the coalition and 0 otherwise.

Suppose as a result of a search operation $\sum_{r=1}^{R} \boldsymbol{S_r'}\ \boldsymbol{A^F}\ \boldsymbol{S_r} = \boldsymbol{A^r}$, (1.21) is satisfied. A coalition $\boldsymbol{C^k}$ is formed by the operation on $\boldsymbol{A^r}$,

$$\theta_k A^r \theta_k' = \sum_{j=1}^{K}\sum_{k=1}^{K} Ajk = \boldsymbol{C^k} \qquad (1.24).$$

Or perhaps more appropriately since $\boldsymbol{C^k}$ is a collection of activities,

$$\theta_k A^r \theta_k' = \bigcup_{i,k\in C^k} \mathrm{A}_{jk} = \boldsymbol{C^k} \qquad (1.24a).$$

In terms of payoffs the choice vector results a coalition of payoffs defined as

$$\theta_k' B^r \theta_k = \sum_{j=1}^{K}\sum_{k=1}^{K} Bjk = \boldsymbol{B^k} \qquad (1.25)$$

That is, each coalition $\boldsymbol{C^k}$ gives rise to potential payoffs

$$\sum_{j=1}^{k}\sum_{k=1}^{k} w_j Ajk = \sum_{j=1}^{K}\sum_{k=1}^{K} mc_k = \boldsymbol{B^k} \qquad (1.25a)$$

Combining search and choice processes together, we can represent the partition of the organization matrix, defined at the fundamental level $\boldsymbol{A^F}$ (consisting of n activities) into a coalition structure

$$\Theta_F \left[\sum_{r=1}^{R} \mathrm{S_r}' \boldsymbol{A^F} \mathrm{S_r}\right]\Theta_F.' = \boldsymbol{A^{F+1}} \qquad (1.26)$$

The coalition structure described by (1.24) consists of a coalition structure of R coalitions each with K activities and 0 ≤ K≤ n). In terms of payoffs, search and choice result in the formation at higher level consisting of a set of R payoffs corresponding to the R coalitions

$$\Theta_F \left[\sum_{r=1}^{R} \mathrm{S_r}' \boldsymbol{B^F} \mathrm{S_r}\right]\Theta_F' = \boldsymbol{B}^{F+1} \qquad (1.26a)$$

Thus through the operations of $\boldsymbol{S}$ and $\boldsymbol{\Theta}$ fundamental activities form coalitions at higher levels of the organization matrix.

The organization matrix as a hierarchy

Alternatively we can rewite the above equations as

$$\Theta_F \boldsymbol{A^r} \Theta_F' = \boldsymbol{A^{F+1}} \qquad (1.27)$$

and

$$\Theta_F \, \boldsymbol{B^r} \, \Theta_F' = \boldsymbol{B^{F+1}} \qquad (1.27a)$$

In general, if we consider an organization matrix at a particular level A^F (a matrix of n fundamental activities), since $\Theta_F = (\theta_1^F, \theta_2^F, ...\theta_k^F, ...\theta_R^F)$, is an n x R matrix (so Θ' is an R x n matrix), we can arrive at a new (higher level) organization matrix, partitioned into R coalitions. The new higher level organization matrix is an (R x R) diagonal matrix, whose diagonal elements are coalitions of lower level activities.

At the higher level new complementarities (synergies) appear, offering new opportunities for coalition formation and higher level coalition structures.

Thus organizations are made up of successively larger building blocks ($\boldsymbol{A^F}$, $\boldsymbol{A^{F+1}}$ ……$\boldsymbol{A^{F+m}}$ and $\boldsymbol{B^F}$, $\boldsymbol{B^{F+1}}$ …..,$\boldsymbol{B^{F+m}}$). Divisionalised (or M form) organizations for example, are coalitions of divisions, which are coalitions of business units and subsidiaries, which in turn are made up of coalitions of projects and so on until we reach the level of fundamental activities or teams. Similarly organizations may themselves form coalitions; alliances, supply chains and so on.

Using an expression that includes both search and decision processes, we have a transformation of A^F through a search process $\boldsymbol{S}$ into A^r and a decision process Θ, into a new, higher level matrix A^{F+1} by the following operations:

$$\Theta_F' \, [\sum_{r=1}^{R} S_r' \, A^F \, S_r] \Theta_F. = A^{F+1} \qquad (1.28)$$

Similarly writing the expression in terms of payoffs we have

$$\Theta_F' \, [\sum_{r=1}^{R} S_r' \, B^F \, S_r \,] \, \Theta_F = \boldsymbol{B^{F+1}} \qquad (1.28a)$$

Initially A^{F+1} and $\boldsymbol{B^{F+1}}$ appear to be diagonal matrices with blocks of fundamental activities forming a coalition structure at level F+1. For example $\boldsymbol{B^{F+1}}$ is a diagonal matrix with B_{kk}^{F+1} as the kth row of the diagonal of the higher level organization matrix. However at that level new complementarities and hence new coalition possibilities may appear.

As figure 1 illustrates, organizations are made up of coalitions within coalitions within coalitions made all made up of building blocks of fundamental activities. The coalition in (f) is made up of a series of sub coalitions and activities.

Generalisation

To elaborate on the notion of hierarchies of the organization matrix, suppose that the that it is defined at M+1 levels, (F, F+1, F+2, ……, F+M), with coalition structures corresponding to each level written, $(L_F^p, L_{F+1}^p, ..., L_{F+M}^p)$ composed respectively of (R^F, R^{F+1},…….., R^{F+M}). Generally, $R^F \geq R^{F+1} \geq R^{F+2} \geq ..... \geq R^{F+M}$, because each successive coalition structure is made up of lower level coalitions. A typical coalition $\boldsymbol{C^k}$ defined at a particular level of the organization matrix can be written $(C_F^k, C_{F+1}^k, ..., C_{F+M}^k)$. Successive coalitions in the hierarchy of the organization matrix are made up of increasing numbers of fundamental activities. If successive coalitions have respectively numbers of fundamental activities (K_F, K_{F+1},….. K_{F+M}), then ($K_F \leq K_{F+1} \leq ..... \leq K_{F+M}$).

Consider F + m as any arbitrary level of the organizational hierarchy (m = 0,1,……, M) We can denote a typical coalition in activity terms as,

$$C_{F+m}^k = \sum_{j,k \in C_F^k} \sum_{j,k \in C_{F+1}^k} ...... \sum_{j,k \in C_{F+M}^k} A_{jk}^F \quad (1.29)$$

Expressing the typical coalition in payoff terms we have

$$B_{F+m}^k = \sum_{j,k \in B_F^k} \sum_{j,k \in B_{F+1}^k} ...... \sum_{j,k \in B_{F+m}^k} B_{jk}^F \quad (1.29a)$$

The typical (potential) coalition structure is written

$$\mathrm{L}_{\mathrm{F+m}}^{\mathrm{p}} = \{\mathrm{C}_{\mathrm{F+m}}^{1}, \mathrm{C}_{\mathrm{F+m}}^{2}, ..., \mathrm{C}_{\mathrm{F+m}}^{\mathrm{k}}, ..., \mathrm{C}_{\mathrm{F+m}}^{\mathrm{R^{F+M}}}\} \quad (1.30)$$

or

$$\mathrm{L}_{\mathrm{F+m}}^{\mathrm{p}} = \{\mathrm{B}_{\mathrm{F+m}}^{1}, \mathrm{B}_{\mathrm{F+m}}^{2}, ..., \mathrm{B}_{\mathrm{F+m}}^{\mathrm{k}}, ..., \mathrm{B}_{\mathrm{F+m}}^{\mathrm{R^{F+M}}}\} \quad (1.30a)$$

Adaptation

Evolution is a response to higher anticipated payoffs. It takes the form of changing coalitions of activities on the organization matrix. Coalitions change for a number of reasons including: (a) discovery effects, (b) efficiency effects, (c) miscalculation effects and (d) distribution effects.

(a) Discovery effects

.

We can think of discovery effects as taking place as a result of changes in the system state of the organization matrix. A change in the system state of the organization matrix is indicated by the addition of new activities or assets, ***A*** or ***B***, changes in the productivity of activities or assets, or by changes in value system underlying ***W***.

(b) Efficiency effects

Changes due to efficiency effects are the result of agents switching coalitions in the search for higher payoffs; recognizing that higher payoffs are potentially attainable within the existing system state by switching coalitions.

(c) Miscalculation effects

Miscalculation effects describe coalition changes that stem from disappointed expectations. They may be associated with bounded rationality; overestimation, either of the asset base, or the potential flow of activities and their contribution to the coalition; disappointed expectations by agents about payoffs. Recognition of miscalculation leads to search for alternatives, and perhaps a switch of coalition allegiance.

(d) *Distribution (renegotiation) effects*

Distribution effects are ubiquitous and form the main rationale for changes in coalitions and coalition structures. That is reformation of coalitions is stimulated by the emergence of a relationship

$$f_k < \boldsymbol{F_k}^* \text{ (some k } \varepsilon \text{ n and some } \boldsymbol{L} \ \varepsilon \ \boldsymbol{L^R}\text{)}$$
$$mc_J \geq 0 \ \ all \ \ C^k \in L^R and (k = 1, 2, ...K) \qquad (1.31)$$

Changes due to pure distribution effects result from realisation by agents that they can earn higher payoffs by switching coalitions or re-negotiating payoffs within existing coalitions. Distribution effects are pervasive. They are also implicit in the other effects. They motivate changes in coalitions and coalition structures

5. AN EXAMPLE

It is useful to consider a simplified example to clarify the issues so far. Suppose n = 4.The organization matrix is written as

$$A^F = \begin{bmatrix} A_{11} A_{21} A_{31} A_{41} \\ A_{12} A_{22} A_{23} A_{24} \\ A_{31} A_{32} A_{33} A_{34} \\ A_{41} A_{42} A_{43} A_{44} \end{bmatrix} \qquad \text{(E.i)}$$

The corresponding payoff matrix is the result of the mapping, $\boldsymbol{W}$**:** $[A_{jk}] \rightarrow [B_{jk}]$ into payoffs $\boldsymbol{B^F}$.

$$WA^F = \begin{bmatrix} w_1 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 \\ 0 & 0 & w_3 & 0 \\ 0 & 0 & 0 & w_4 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} = \boldsymbol{B^F}$$

$$\boldsymbol{B^F} = \begin{bmatrix} B_{11} B_{21} B_{31} B_{41} \\ B_{12} B_{22} B_{23} B_{24} \\ B_{31} B_{32} B_{33} B_{34} \\ B_{41} B_{42} B_{43} B_{44} \end{bmatrix} \qquad \text{(E.ii)}$$

Consider a possible partition into one coalition containing activities {1 2 and 3}, ($\boldsymbol{S_1}$), and another ($\boldsymbol{S_2}$) consisting of activity {4}.

$$\boldsymbol{S_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and } \boldsymbol{S_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(E.iii)}$$

Performing the appropriate search operations gives

$$\boldsymbol{S_1' A^F S_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_{11} A_{21} A_{31} A_{41} \\ A_{12} A_{22} A_{23} A_{24} \\ A_{31} A_{32} A_{33} A_{34} \\ A_{41} A_{42} A_{43} A_{44} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} A_{11} A_{21} A_{31} 0 \\ A_{12} A_{22} A_{23} 0 \\ A_{31} A_{32} A_{33} 0 \\ 0 \quad 0 \quad 0 \quad 0 \end{bmatrix} \qquad \text{(E.iv)}$$

The second coalition consists of a single activity A_{44}, the resulting partition is

$$\boldsymbol{S_2' A^F S_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_{11} A_{21} A_{31} A_{41} \\ A_{12} A_{22} A_{23} A_{24} \\ A_{31} A_{32} A_{33} A_{34} \\ A_{41} A_{42} A_{43} A_{44} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} \end{bmatrix} \qquad \text{(E. iv(a))}$$

Thus we have a potential coalition structure ***L*** denoted by the partition $\boldsymbol{A^r}$, such that

$$\boldsymbol{S_1' A^F S_1 + S_2' A^F S_2 = A^r} \qquad \text{(E.v)}$$

Where

$$\boldsymbol{A^r} = \begin{bmatrix} A_{11}A_{21}A_{31} & 0 \\ A_{12}A_{22}A_{23} & 0 \\ A_{31}A_{32}A_{33} & 0 \\ 0 \quad 0 \quad 0 & A_{44} \end{bmatrix} \qquad \text{(E.v(a))}$$

Similarly we have a partitioned matrix $\boldsymbol{B^r}$

$$\boldsymbol{B^r} = \begin{bmatrix} B_{11}B_{21}B_{31} & 0 \\ B_{12}B_{22}B_{23} & 0 \\ B_{31}B_{32}B_{33} & 0 \\ 0 \quad 0 \quad 0 & B_{44} \end{bmatrix} \qquad \text{(E.v(b))}$$

Given 4 activities there are 52 possible coalition structures in $\boldsymbol{L^p}$. Search matrices $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$ above partition ***A*** (or ***B***) into a particular structure, ***L***, of two coalitions (one consisting of three activities and another of a single activity). Suppose that feasibility conditions are satisfied for ***L***: the value of both coalitions is positive and (ii) payoffs to every activity is in each coalition is positive, and (iii) the sum of the payoffs to activities in each of the coalitions does not exceed the value of the coalition. That is

$$\begin{aligned} &\sum_{j,k \in C^1} Bjk \geq 0 \quad (j,k = 1,2,3) \\ &for\, C^1 and \;\; for \;\; C^2 \\ &B_{44} \geq 0 \\ \\ &and \\ &f_k \;\; \geq 0 \quad (k = 1,2,3,4) \\ &\sum_{k=1}^{3} f_k \leq B^1 \;\; and \;\; f_4 \geq 0 \end{aligned} \qquad \text{(E.vi)}$$

Suppose the choice criteria are satisfied for ***L***

$$f_k \geq F_{p/} \ (k = 1,2,3,4)$$
$$mc_k \geq 0 \quad all\ in\ C^1 and\ C^2 \qquad \text{(E.vii)}$$

So $\boldsymbol{L}$ is the chosen coalition structure and $\boldsymbol{C^1}$ and $\boldsymbol{C^2}$ are the chosen coalitions. So via the operator Θ, $\boldsymbol{L^F}$ becomes the chosen coalition structure, size $\boldsymbol{R^F}$ ($\boldsymbol{R^F}$= 2).

$$\Theta' = \begin{bmatrix} 1110 \\ 0001 \end{bmatrix} \qquad \text{(E.viii)}$$

Adopting the choice operator Θ on $\boldsymbol{A^r}$ and $\boldsymbol{B^r}$ respectively results in a higher level organization matrix $\boldsymbol{A^{F+!}}$ and $\boldsymbol{B^{F+!}}$

$$\Theta' A^r \Theta = \begin{bmatrix} \sum_{j,k=1}^{3} Ajk & 0 \\ 0 & A_{44} \end{bmatrix} = A^{F+1}$$
$$that\ is\ given\ new\ complementarities \qquad \text{(E.ix)}$$
$$A^{F+1} = \begin{bmatrix} A_{11}^{F+1} & A_{12}^{F+1} \\ A_{21}^{F+1} & A_{22}^{F+1} \end{bmatrix}$$

Suppose as a result of search at level F+1 (via the search matrix $\boldsymbol{S^{F+1}}$) feasibility and choice conditions are satisfied given the emerging complementarities. Writing the search matrix as

$$S^{F+1'} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{(E.x)}$$

We have in this case of a grand coalition at level F+1, a situation in which $\boldsymbol{A^{F+1}} = \boldsymbol{A^{r+1}}$ and $\boldsymbol{B^{F+1}} = \boldsymbol{B^{r+1}}$.

$$A^{F+2} = \begin{bmatrix} A_{11}^{F+2} \end{bmatrix} \qquad \text{(E.xi)}$$

And

$$B^{F+2} = \begin{bmatrix} B_{11}^{F+2} \end{bmatrix} \qquad \text{(E.xi(a))}$$

Transforming the organization matrix into a payoff landscape

Consider the following organization matrix B in payoff terms

$$B^F = \begin{bmatrix} 1 & 2 & 3 & -16 \\ 6 & 5 & 4 & 13 \\ 7 & 8 & 9 & -11 \\ -15 & 14 & 12 & 10 \end{bmatrix} \tag{E.xii}$$

We calculate marginal values and Shapley weights corresponding to (E2.1) as follows in table 1

Weights	**Marginal values**			
$\frac{(k-1)!(n_k-k)!}{n_k!}$	m_1	m_2	m_3	M_4
$\frac{3!0!}{4!} = \frac{1}{4}$	v (1234) – (234) (52 – 64) = - 12	v (1234) – v(134) (52 – 0) = 52	v (1234) – v(124) (52 -20) = 32	v (1234) – v(123) (52 – 45) = 7
$\frac{2!1!}{4!} = \frac{1}{12}$	v (123) – v(23) (45 – 26) = 19 v (134) – v(34) (0 – 20) = -20 v (124) – v(24) (20 -42) = -22	v (123) – v(13) (45 -20) =25 v (124) – v(14) (20 +20) = 40 v (234) – v(34) (64 -20) = 44	v (123) – v(12) (45 -14) = 31 v (134) – v(14) (0 +20) = 20 v (234) – v(24) (64 -0) = 22	v (134) – v(13) (0 – 20) = -20 v (124) – v(12) (20 -14) = 6 v (234) – v(23) (64 – 26) = 38
$\frac{1!2!}{4!} = \frac{1}{12}$	v (12) – v(2) (14 -5) = 9 v (13) – v(3) 20 – 9= 11 v (14) – v(4) (-20 -10) = -30	v (12) – v(1) (14 -1)= 13 v (23) – v(3) (26 – 9) = 17 v (24) – v(4) (42 – 10) = 32	v (13) – v(1) 20 – 1 = 19 v (23) – v(2) (26 – 5) = 21 v (34) – v(4) (20 -10) = 10	v (14) – v(1) (-20 – 1) = -21 v (24) – v(2) (42 -5) = 27 v (34) – v(3) (20 -9) = 11
$\frac{0!3!}{4!} = \frac{1}{4}$	v(1) 1	v(2) 5	v (3) 9	v (4) 10

Table 1

In Table 3 we set out the corresponding coalitions and coalition structures corresponding to (E2.1), together with the membership of coalitions contained in the respective structures and the marginal values of activities to the coalitions within a structure. Note that the number of coalition structures for n = 4 (that is L* = 52).

6. THE TRANSITION FRONTIER

We focus on search in this section and upon the tendency under ergodic search for coalitions on the organization matrix to gravitate to a state of self ordered criticality, illustrated by set of possibility frontiers on which change on all scales is possible.

Varieties of search

Table 2 presents a classification of search procedures in relation to the organization matrix. Focus may be on the matrix itself or upon decisions. Ergodic search is such that each one of the 2^N possible coalitions is equally likely. We might think of ergodic search as making every decision associated with a coalition equally likely, giving $2^{N(N-1)}$ configuration. Alternatively we may think in terms of restricted search. In which search is governed by the nature of a fitness landscape which is decomposed by restricting the extent of interdependence between activities. Alternatively search and choice may be restricted by what an organizational grammar, a set complex set of interacting rules that govern decisions.

	Ergodic search	Directed search
Matrix [Aik]	Statistical Mechanics **1**	Decomposition **2**
Decisions (θi θk)	Spin glass **3**	**4** Organizational Grammar

Table 2: Varieties of search.

a. Ergodic searches in the statistical mechanics formulation, focus on the organization matrix itself. Coalitions will spend most time in structure with the highest probability. Thus in the simple four activity situation illustrated in Table 1 if all 16 possible coalitions are equally likely, a coalition structure of two member coalitions is most likely to occur. In the absence of another dynamic, the coalition structure is likely to gravitate to this, the most probable.
b. We may view the organization matrix as a Kauffman type NK landscape in which the size of the matrix, N and the number of connections, K are tuning parameters that control the extent of search. As K increases so does the interdependence and the ruggedness of the matrix: high K meaning that changes in one coalition have knock on effects that are distributed throughout the landscape making search increasingly difficult as K increases. High K means unreliable information and increasing risk associated with change[20]. In Kauffman's view evolution unlikely with high

[20] In Kauffman's systems evolution takes place at the edge of chaos, the cusp between order and chaos at K=3 (in his terms connectivity to itself plus no more than two other activities).

interdependence (high K). Evolution is most likely with low K (K approximately 3) which as a borderline between order and chaos becomes a critical connectivity to which evolutionary systems gravitate. However there is no mechanism in Kauffman that explains why this should happen.

c. Spin glasses are a recent application of statistical mechanics. If decisions (θiθk) are treated as random magnetic spins, as different coalitions are formed payoffs change abruptly (phase transitions). A spin glass system evolves by *flipping* from one coalition structure to another. Two essential ingredients are frustration and randomness, leading to many possible structures. Coalitions may become trapped in a basin of attraction: either c*haotically* moving from one coalition state to another or *frozen* into any one of many states. In either case, realised payoffs (and coalitions) will be randomly determined.

d. Organizational grammar describes the rules, laws, treaties, agreements, culture, traditions and conventions that govern search choice and adaptation on the organization matrix. It originates both outside and inside of organizations, and in mind sets of agent. It determines which payoffs the organization focuses on and which stakeholders are considered most important. Organizational grammar has any number of dimensions; formal/informal, personal/social and internal/external with respect to the organization.

3 transition frontiers

Focusing on ergodic strategies we now open the discussion of the gravitation coalitions and coalition structures to a possibility frontier, an attractor on which change on all scales is possible. We approach the issue from three different aspects: (a) coalition size (b) quitting coalitions and (c) coalition structures.

Coalition size

Suppose we have an organization matrix of n activities and the probability of joining a coalition p (or not) is ½. The distribution of relative coalition size is given by the positive binomial expression

$$\{p^n, np^n, n(n-1)/2!n^n, n(n-1)(n-2)/3!p^n, \ldots\ldots\ldots\ldots\ldots\ldots, p^n\} = \sum_{s=0}^{n} \binom{n}{s} p^n .^{21} \qquad (1.32)$$

This expression (for increasing n) is summarised by the Pascal triangle. As n increases, the binomial distribution approaches the normal, with mean coalition size np and variance np^2. Thus as n increases the probability of forming a coalition of a given size s increases. Similarly as we increase s, the coalition size for a given n, the distribution of coalitions by size follows the binomial series. For the 8 activities in Figure 1, 2^8 possible

[21] Where $\binom{n}{s} = \frac{n!}{s!(n-s)!}$

coalitions and the probability of coalitions of size 8, 7, 6, 5….,1, 0 follows the binomial expression; that is $2^{-8}\{1,8,28,56,70,........,8,1\}$.

The possibility frontier relates to the probability distribution of coalition size and approximates to the bell shape of normal distribution. Alternatively we can see it in terms of the probability of a coalition of a given size s ($P\{C^s\}$)as n the number of activities increases which is also described by the binomial distribution: the probability of a coalition of exactly s activities being,

$$P\{C^s\} = \binom{n}{s} p^n \qquad (1.33)$$

Quitting coalitions

Given the assumptions, the probability of s activities quitting a coalition of size K is given by the negative binomial distribution: it is the probability of s-1 activities quitting a coalition of size K-1, time the probability an additional activity quitting a coalition of size K; write this as $Q\{C^K_{s\,quitting}\}$ and $Q\{C^K_{s\,quitting}\} = \binom{K-1}{s-1} q^{K-1}.q = \binom{K-1}{s-1} q^K$

(1.34)

It is useful to see the possibility frontier in this case as describing the probability of an entire coalition of size K breaking up, and to view the behaviour of the resulting distribution as K increases. In this case, K = s. so $\binom{K-1}{s-1}$ = 1 and the possibility frontier is described by Figure 4: the probability that an entire coalition of size K will break up falls asymptotically with K.

$$Q\{C^K_{s=K\,quitting}\} = \frac{d}{dK} q^k = q^k \,In\, q \prec 0 \text{ and } \frac{d^2}{dK^2} = q^k\,(In\, q)^2 \succ 0 \text{ since q< 1.[22]} \qquad (1.35)$$

[22] Note In q for q = 0.5 is – 0.69315 and q^K declines with k.

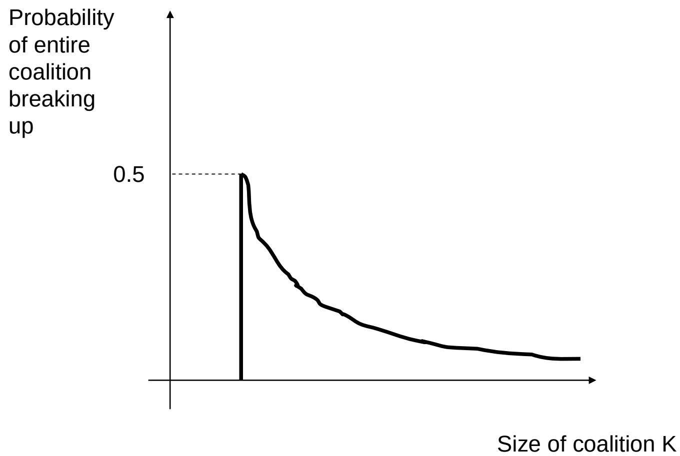


Figure 4

Coalition structures and the transition frontier

The possibility frontier can also be cast in terms of coalition structures. Recalling (1.16) a coalition structure ***L*** describes a particular partition of n activities into a set of R coalitions

$$\boldsymbol{L} = \{\boldsymbol{C}^1, \boldsymbol{C}^2, \ldots, \boldsymbol{C}^k, \ldots, \boldsymbol{C}^R\} \qquad (1.16).$$

Viewing a coalition structure a macrostate corresponding to an ensemble of coalitions, or microstates we can rewrite (1.16) as

$$\boldsymbol{L} = \frac{n!}{n_1!, n_2! \ldots n_R!} = \frac{n\,!}{\prod_{r=1}^{R} n_r\,!} \qquad (1.36)$$

Or in logarithms as

$$In\ L = In\ n! - \sum_r n_r! = n\, In\, n - n - \sum n_\partial\, In\, n_r - \sum_r n_r$$

$$L = In\left(\frac{n!}{\prod_{r=1}^{R} n_r!}\right) = In\, n! - In \prod_r n_r! \qquad (1.36a)$$

$$= In\ n! - \sum_r n_r!$$

i.

Using Stirling's approximation (*log n! ≈ n log n – n*) and the constraint $\sum_r n_r = n$ we can rewrite (1.36a) as [23]

$$In\ L = n\, In\, n - \sum_r n_r\, In\, n_r \qquad (1.37)$$

Expression (1.37) is the foundation of the possibility frontier in Figure 1, which we reproduce in more detail as Figure 5. If n_r = n the probability of that structure defined as In ***L*** on the ergodic hypothesis is zero (In L = 0). As In n_r falls towards zero ($n_r \rightarrow 1$) the probability of the particular coalition structure rises. The shape of the curve follows from the derivatives; $\frac{d\left(InL\right)}{dn_r} \prec 0\, and\, \frac{d^2\left(InL\right)}{dn_r^{\,2}} \succ 0$.[24]

[23] Using Sterling's approximation $In\ L = In\ n! - \sum_r n_r! = n\, In\, n - n - \sum n_\partial\, In\, n_r - \sum_r n_r$

[24] $\frac{d\left(InL\right)}{dn_r} = -(n_r^{-1} + In\, n_r); \frac{d^2\left(InL\right)}{dn_r^{\,2}} = (n_r^{-1} - n_r^{-2})$.

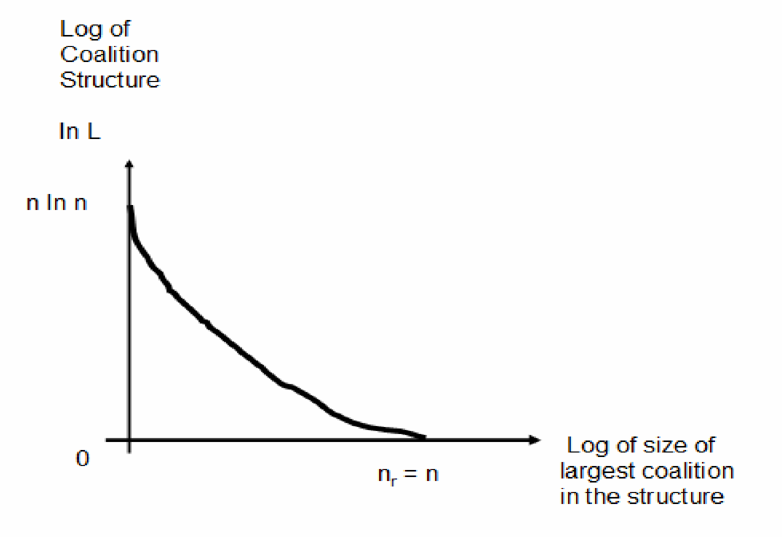


Figure 5

7. CONCLUSING REMARKS

Possibly crisis is the natural state of affairs. We are faced now as usual with major crises, social, economic, political and personal. Perhaps change itself is the crisis, opening up desire, opportunity, aspiration that we try to direct and limit through designing grammar that brings the illusion of permanence, to channel desire and opportunity into social and organizational evolution and to veil entropy, decay and impermanence, the other face of change. In some traditions desires, aspirations and change are associated with progress, in others with sorrow (Kalansuriya, 1987).

Organization matrix theory culminates in a state of self ordered criticality. The theory cannot possibly be a general theory in the same way thermodynamics or Newtonian mechanics are general theories or even the models of economics. Rather, the paper presents a group of concepts that enable lot can be learned about the way organizations evolve and out of which general models have been constructed that provide practical and useable frameworks for businesspeople. They may present the beginnings of a new paradigm or perhaps a resuscitation of an older one in which distribution and demand play primary roles. vehicles for successful research. Maybe the theory has heuristic value opening new doors up new ways of thinking and encourages deconstruction of reasoning that has become a barrier to understanding.

On the idea of self ordered criticality contained in the model and the assertion that society has reached such a point, two possible scenarios arise. The first is an Orwellian 1984 scenario; a brief life for the global market economy, from which emerges a new form of monopoly capitalism dominated by coalitions of nation states and corporations that operate as supra national states, with private armies combining with the *chaos* mentioned above in a scenario centred on rivalry.

Another scenario stems from the second industrial revolution which we are now experiencing: machines (that in the first revolution replaced manual tasks) are now replacing intelligent cognitive tasks that were formerly associated exclusively with human beings. This raises the question; *What, if anything, is special about human beings?*

REFERENCES

On request