

Modelling Naira/Dollar Exchange Rate Volatility: Application Of Garch And Assymmetric Models

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This paper investigated the volatility of Naira/Dollar exchange rates using GARCH (1,1), GJR-GARCH(1,1), EGARCH(1,1), APARCH(1,1), IGARCH(1,1) and TS-GARCH(1,1) models. The TS-GARCH and APARCH models are the best models as they have all the parameters of the variance equations being significant. However, all the models except EGARCH model, show that there is high volatility persistence in Nigerian foreign exchange market with all variances being non-stationary. The APARCH model and GJR-GARCH model for the managed floating rate regime show the existence of statistically significant asymmetry effect. All the asymmetry models rejected the hypothesis of leverage effect.

1. Introduction

Prior to the introduction of structural adjustment programme in Nigeria in 1986, the country adopted a fixed exchange rate regime supported by exchange control regulations that engendered significant distortions in the economy. The country depends heavily on imports from various countries as most industries in Nigeria import their raw materials from foreign countries. Apart from raw materials, there were massive importation of finished goods with the adverse consequences for domestic production, balance of payments position and the nation's external reserves level (Sanusi, 2004). The foreign exchange market in the fixed exchange rate period was characterized by high demand for foreign exchange which can not be adequately met with the supply of foreign exchange by the Central Bank of Nigeria (CBN). The fixed exchange rate period was also characterised by sharp practices perpetrated by dealers and end-users of foreign exchange (Sanusi, 2004). The inadequate supply of foreign exchange by the CBN promoted the parallel market for foreign exchange and created uncertainty in foreign exchange rates. The introduction of SAP in Nigeria in September 1986 which deregulated the foreign exchange market led to the introduction of market determined exchange rate, managed floating rate regime. The CBN usually intervene in foreign exchange market through its monetary policy actions and operations in the money

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market to influence the exchange rate movement in the desired direction such that it ensures the competitiveness of the domestic economy. This introduction of managed floating rate regime tends to increase the uncertainty in exchange rates, thus, increasing the volatility of exchange rate by the regime shifts. This made the exchange rate to be the most important asset price in the economy. The understanding of the behaviour of exchange rate behaviour is important to monetary policy (Longmore and Robinson, 2004). The exchange rate has been found to be an important element in the monetary transmission process [Robinson and Robinson (1997), Allen and Robinson (2004)] and movements in this price has a significant pass-through to consumer prices (see Robinson (2000a and 2000b) and McFarlane (2002))

According to Longmore and Robinson (2004), because of the thinness and volatility of the market, the policy makers focus on the information content of the short-term volatility especially in deciding intervention policy. The uncertainty of the exchange rate shows how much economic behaviors are not able to perceive the directionality of the actual or future volatility of exchange rate, that is, it is a different concept from the volatility of the exchange rate itself in that it means that the more forecast errors of economic behaviors made, the higher the trends in the uncertainty of the exchange rate are shown (Yoon and Lee, 2008). The volatility of financial assets has been of growing area of research (see Longmore and Robinson (2004) among others). The traditional measure of volatility as represented by variance or standard deviation is unconditional and does not recognize that there are interesting patterns in asset volatility; e.g., time-varying and clustering properties. Researchers have introduced various models to explain and predict these patterns in volatility. Engle (1982) introduced the autoregressive conditional heteroskedasticity (ARCH) to model volatility. Engle (1982) modeled the heteroskedasticity by relating the conditional variance of the disturbance term to the linear combination of the squared disturbances in the recent past. Bollerslev (1986) generalized the ARCH model by modeling the conditional variance to depend on its lagged values as well as squared lagged values of disturbance, which is called generalized autoregressive conditional heteroskedasticity (GARCH). Since the work of Engle (1982) and Bollerslev (1986), various variants of GARCH model have been developed to model volatility. Some of the models include IGARCH originally proposed by Engle and Bollerslev (1986), GARCH-in-Mean (GARCH-M) model introduced by Engle, Lilien and Robins (1987), the standard deviation GARCH model introduced by Taylor (1986) and Schwert (1989), the EGARCH or Exponential GARCH model proposed by Nelson (1991), TARARCH or Threshold ARCH and Threshold GARCH were introduced independently by Zakoian (1994) and Glosten, Jaganathan, and Runkle (1993), the Power ARCH model generalised by Ding, Zhuanxin, C. W. J. Granger, and R. F. Engle (1993) among others.

The modeling and forecasting of exchange rates and their volatility has important implications for many issues in economics and finance. Various family of GARCH models have been applied in the modeling of the volatility of exchange rates in various countries. Taylor (1987) and more recently West and Chow (1995) examined the forecast ability of exchange rate volatility using a number of models including ARCH using five U.S. bilateral exchange rate series. They found that generalised ARCH (GARCH) models

were preferable at a one week horizon, whilst for less frequent data, no clear victor was evident. Some other studies on the volatility of exchange rates include Meese and Rose (1991), McKenzie (1997), Christian (1998), Longmore and Wayne Robinson (2004), Yang(2006) Yoon and Lee (2008) among others. Little or no work has been done on modeling exchange rate volatility in Nigeria particularly using GARCH models. This paper attempts to fill this gap. The exchange rate volatility has implications for many issues in the arena of finance and economics. Such issues include impact of foreign exchange rate volatility on derivative pricing, global trade patterns, countries balance of payments position, government policy making decisions and international capital budgeting. The purpose of this paper is to model exchange rate volatility in Nigeria using family of GARCH models. The paper will examine the volatility and asymmetry of exchange rates in Nigeria using GARCH, TARARCH, EGARCH, the standard deviation GARCH, Power ARCH and IGARCH models. The deregulation of foreign exchange market in Nigeria in September 1986 could have affected the volatility of exchange rates in Nigeria. This paper, apart from presenting full sample results, will separate present the results of volatility in a fixed exchange rate regime and floating exchange rate regime. The rest of this paper is organised as follows: Chapter two discusses Theoretical background and literature review on models of time varying volatility while Chapter three discusses methodology. The results are presented in Chapter four while concluding remarks are presented in Chapter five.

2. Literature Review On Models Of Time Varying Volatility

The need of long lag to improve the goodness of fit when we adopt the autoregressive conditional heteroskedasticity (ARCH) model occurs at times. To overcome this problem, Bollerslev (1986) suggested the generalized ARCH (GARCH) model, which means that it is a generalized version of ARCH. The GARCH model considers conditional variance to be a linear combination between squared of residual and a part of lag of conditional variance. This simple and useful GARCH is the dominant model applied to financial time series analysis by the parsimony principle. GARCH (1,1) model can be summarized as follows:

$$s_t = b_0 + \varepsilon_t \quad \varepsilon_t / \phi_{t-1} \sim N(0, \sigma_t^2) \quad (1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (2)$$

where, σ^2 is conditional variance of ε_t and $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$. Equation (2) shows that the conditional variance is explained by past shocks or volatility (ARCH term) and past variances (the GARCH term). Equation (2) will be stationary if the persistent of volatility

shocks, $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j$ is lesser than 1 and in the case it comes much closer to 1, volatility

shocks will be much more persistent. As the sum of α and β becomes close to unity, shocks die out rather slowly (see Bollerslev (1986)). To complete the basic ARCH specification, we require an assumption about the conditional distribution of the error term. There are three assumptions commonly employed when working with ARCH

models: normal (Gaussian) distribution, Student's t -distribution, and Bollerslev (1986, 1987), Engle and Bollerslev (1986) suggest that GARCH(1,1) is adequate in modeling conditional variance. The GARCH model has a distinctive advantage in that it can track the fat tail of asset returns or the volatility clustering phenomenon very efficiently (Yoon and Lee, 2008). The normality assumption for the error term in (1) is adopted for most research papers using ARCH. However, other distributional assumptions such as Student's t -distribution and General error distribution can also be assumed. Bollerslev (1987) claims that for some data the fat-tailed property can be approximated more accurately by a conditional Students-distribution.

If in Equation (1), sum of α and β is equal to 1, then shocks to volatility persist forever, and the unconditional variance is not determined by the model. Engle and Bollerslev (1986) call this type of process 'Integrated-GARCH'. They call this model the integrated GARCH' or IGARCH' model. The IGARCH model is, thus, given as follows:

$$\sigma_t^2 = \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (3)$$

such that

$$\sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 = 1 \quad (4)$$

The impacts of variance shocks remain forever in the IGARCH model, in contrast to the stationary variance case. A weakness of the GARCH model is that the conditional variance is merely dependent on the magnitude of the previous error term and is not related to its sign. It does not account for skewness or asymmetry associated with a distribution. Thus, GARCH model can not reflect leverage effects, a kind of asymmetric information effects that have more crucial impact on volatility when negative shocks happen than positive shocks do (Yoon and Lee, 2008). Because of this weakness of GARCH model, a number of extensions of the GARCH (p, q) model have been developed to explicitly account for the skewness or asymmetry. The popular models of asymmetric volatility includes, the exponential GARCH (EGARCH) model, Glosten, Jaganathan, and Rankle (1992) GJR-GARCH model, asymmetric power ARCH (APARCH), Zakoian (1994) threshold ARCH (TARCH). The TS-GARCH advanced by Taylor (1986) and Schwert (1990), Ding, Zhuangxin, C. W. J. Granger, and R. F. Engle (1993) generalized power ARCH model, the generalized version of Higgins and Bera (1992) non-linear ARCH (NGARCH) among others.

The TS-GARCH model developed by Taylor (1986) and Schwert (1990) is a popular model used to capture the information content in the thick tails, which is common in the return distribution of speculative prices. The specification of this model is based on standard deviations and is as follows:

$$\sigma_t = \omega + \sum_{i=1}^p \alpha_i |\varepsilon_{t-i}| + \sum_{j=1}^q \beta_j \sigma_{t-j} \quad (5)$$

The GJR-GARCH (p, q) model was introduced by Glosten, Jagannathan and Runkle (1993) to allow for asymmetric effects. The model is given as:

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{k=1}^r \gamma_k \varepsilon_{t-k}^2 \Gamma_{t-k}^- \quad (6)$$

where Γ_t^- (a dummy variable) = 1 if $\varepsilon_t < 0$ and 0 otherwise.

In the GJR-GARCH model, good news $\varepsilon_{t-i} > 0$ and bad news, $\varepsilon_{t-i} < 0$, have differential effects on the conditional variance; good news has an impact of α_i while bad news has an impact of $\alpha_i + \gamma_i$. If $\gamma_i > 0$, bad news increases volatility, and there is a *leverage effect* for the i -th order. If $\gamma \neq 0$, the news impact is asymmetric (see Glosten, Jagannathan and Runkle (1993)). The exponential GARCH (EGARCH) model advanced by Nelson (1991) is the earliest extension of the GARCH model that incorporates asymmetric effects in returns from speculative prices. The EGARCH model is defined as follows:

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^p \alpha_i \left[\frac{\varepsilon_{t-i}}{\sigma_{t-i}} - \sqrt{\frac{2}{\pi}} \right] + \sum_{j=1}^q \beta_j \log(\sigma_{t-j}^2) + \sum_{k=1}^r \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \quad (7)$$

where ω , α_i , β_j and γ_k are constant parameters. The EGARCH(p,q) model, unlike the GARCH (p, q) model, indicates that the conditional variance is an exponential function, thereby removing the need for restrictions on the parameters to ensure positive conditional variance. The asymmetric effect of past shocks is captured by the γ coefficient, which is usually negative, that is, ceteris paribus positive shocks generate less volatility than negative shocks (Longmore and Robinson, 2004). The leverage effect can be tested if $\gamma < 0$. If $\gamma \neq 0$, the news impact is asymmetric. The asymmetric power ARCH (APARCH) model of Ding, Granger and Engle (1993) also allows for asymmetric effects of shocks on the conditional volatility. Unlike other GARCH models, in the APARCH model, the power parameter of the standard deviation can be estimated rather than imposed, and the optional γ parameters are added to capture asymmetry of up to order r . The APARCH (p, q) model is given as:

$$\sigma_t^\delta = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^\delta \quad (8)$$

where $\delta > 0$, $|\gamma_i| \leq 1$ for $i = 1, \dots, r$, $\gamma_i = 0$ for all $i > r$, and $r \leq p$

If $\gamma \neq 0$, the news impact is asymmetric.

The introduction and estimation of the power term in the APARCH model is an attempt to account for the true distribution underlying volatility. The idea behind the introduction of a power term arose from the fact that, The assumption of normality in modeling financial data, which restricts d to either 1 or 2, is often unrealistic due to significant

skewness and kurtosis(Longmore and Robinson, 2004).. Allowing d to take the form of a free parameter to be estimated removes this arbitrary restriction.

3. Methodology

3.1 The Data

The time series data used in this analysis was the average monthly Naira/Dollar exchange rate from January 1970 to December 2007 obtained from various issues of the statistical bulletin and annual report of the Central Bank of Nigeria. The 2007 data was downloaded from the website of the Central Bank of Nigeria. In this study, the return on exchange rate is defined as:

$$r_t = \log\left(\frac{e_t}{e_{t-1}}\right) \quad (9)$$

where e_t mean Naira/dollar exchange rate at time t and e_{t-1} represent naira exchange rate at time $t-1$.

The r_t of Equation (9) will be used in investigating the volatility of exchange rate in Nigeria over the period, 1970 - 2007. The foreign exchange market was deregulated in Nigeria in September 29, 1986. To examine the impact of the deregulation on volatility, results will be presented separately for the period before deregulation, Fixed exchange rate period (January 1970- August 2006) and managed float regime (September 2006 - December 2007).

3.2 Properties Of The Data

The summary statistics of the exchange rate return series is given in Table 1. The mean return for the full sample, pre-deregulation era and deregulation are 0.0112, 0.0031 and 0.0175 respectively while their standard deviations are 0.095, 0.0265 and 0.1242 respectively. The standard deviation appears to be higher after the market has been deregulated following the introduction of market determined exchange rates. The skewness for the full sample and the two sub periods (Fixed rate and Managed floating rate regimes) are 11.6686, 0.531and 9.1498 respectively. This shows that the distribution is positively skewed relative to the normal distribution (0 for the normal distribution). This is an indication of a non symmetric series. The kurtosis for the full sample and the two sub periods (Fixed rate and Managed floating rate regimes) are very much larger than 3, the kurtosis for a normal distribution. Skewness indicates non-normality, while the relatively large kurtosis suggests that distribution of the return series is leptokurtic, signaling the necessity of a peaked distribution to describe this series. This suggests that for the exchange rate return series, large market surprises of either sign are more likely to be observed, at least unconditionally. The Ljung-Box test Q statistics for the full sample and the two sub periods (Fixed rate and Managed floating rate regimes) are all insignificant at the 5% for all reported lags confirming the absence

of autocorrelation in the exchange rate return series. Jarque-Bera normality test rejects the hypothesis of normality for the full sample and the two sub periods (Fixed rate and managed floating rate regimes). Figures 1, 2 and 3 shows the quantile-quantile plots of the exchange rate return for the for the full sample and the two sub periods (Fixed rate and Managed floating rate regimes). Figures 1, 2 and 3 clearly show that the distribution of the exchange rate return series show a strong departure from normality.

The usual method of testing for testing for conditional homoscedasticity by calculating the autocorrelation of the squared return series might not be appropriate here in view of the non-normality of the exchange rate return series (see Mckenzie (1997)). According to Mckenzie (1997), volatility clustering is by no means unique to the squared returns of an assets price. In general, the absolute changes in an assets price will exhibit volatility clustering and the inclusion of any power term acts so as to emphasise the periods of relative tranquility and volatility by highlighting the outliers in that series. It is possible to specify any power term to complete this task from a myriad of options inclusive of any positive value (Mckenzie, 1997). The common use of a squared term is most likely a reflection of the normality assumption made regarding the data. If a data series is normally distributed, then we are able to completely characterise its distribution by its first two moments. As such, it may be appropriate to focus on a squared term. However, if we accept that the data has a non-normal error distribution, then one must transcend into the realm of the higher moments of skewness and kurtosis to adequately describe the data. In this instance, the intuitive appeal of a squared term is lost and other power transformations may be more appropriate (Mckenzie, 1997). Following, Mckenzie (1997), the test for conditional homoscedasticity was carried out by calculating the autocorrelation of power transformed exchange rate return series using powers of 0.25, 0.5 and 0.75. The Ljung-Box $Q^{0.25}$ and $Q^{0.5}$ statistics for the full sample and the two sub periods (Fixed rate and Managed floating rate regimes) are significant at the 5% for all reported lags confirming the presence of heteroscedasticity in the exchange rate return series. The Ljung-Box $Q^{0.75}$ statistics are significant at the 5% level for the full sample and first sub-period (fixed exchange rate period). However, the Ljung-Box test $Q^{0.75}$ statistics are insignificant at the 5% level for all lags for the second sub-period. In view of the insignificance of the Ljung-Box $Q^{0.25}$ and $Q^{0.5}$ test statistics for the second sub-period, it will be safer to reject conditional homoscedasticity for this sub period too.

Table 3 shows the results of unit root test for the exchange rate return series. The Augmented Dickey-Fuller test and Phillips-Perron test statistics for the exchange rate return series are less than their critical values at the 1%, 5% and 10% level. This shows that the exchange rate return series has no unit root. Thus, there is no need to difference the data. In summary, the analysis of the exchange rate return indicates that the empirical distribution of returns in the foreign exchange rate market is non-normal, with very thick tails for the full sample and the two sub periods (Fixed rate and managed floating rate regimes). The leptokurtosis reflects the fact that the market is characterised by very frequent medium or large changes. These changes occur with greater frequency than what is predicted by the normal distribution. The empirical distribution confirms the presence of a non-constant variance or volatility clustering.

Table 1: Summary Statistics and Autocorrelation of the Raw Exchange Rate Return Series

	Full Sample	Fixed rate regime	Managed Float
Summary Statistics			
Mean	0.011	0.003	0.018
Standard Deviation	0.095	0.027	0.124
Skewness	11.669	0.531	9.150
Kurtosis	156.958	19.709	93.834
Jarque-Bera	459695.000 (0.000)*	2324.390 (0.000)*	91581.100 (0.000)*
Observation	455	199	256
Ljung-Box Q Statistics			
Q(1)	0.025 (0.870)	4.555 (0.033)*	0.079 (0.779)
Q(6)	1.587 (0.950)	26.876 (0.000)*	2.061 (0.914)
Q(12)	2.485 (1.000)	30.396 (0.002)*	2.410 (0.998)
Q(20)	2.794 (1.000)	34.579 (0.022)*	2.819 (1.000)

Notes: p values are in parentheses. * indicates significance at the 5% level

Figure 1: Quantile-Quantile Plot of Exchange Rate Return Series Based on the Full Sample (January 1970 – December 2007)

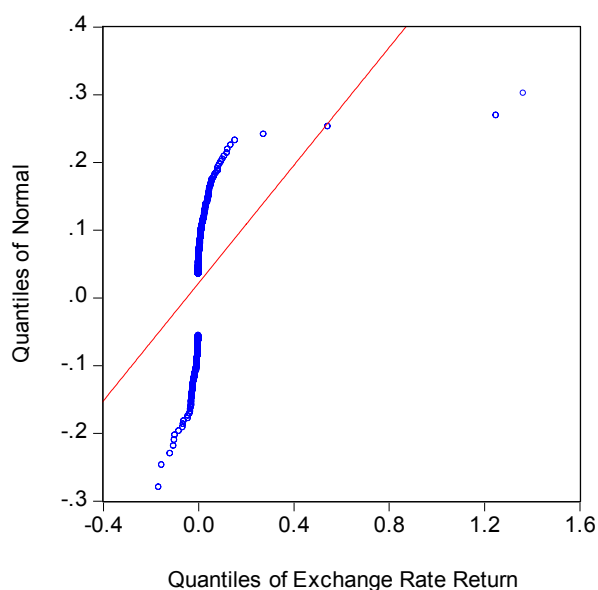


Figure 2: Quantile-Quantile Plot of Exchange Rate Return Series Based for the Fixed Exchange Rate Regime (January 1970 – August 1986)

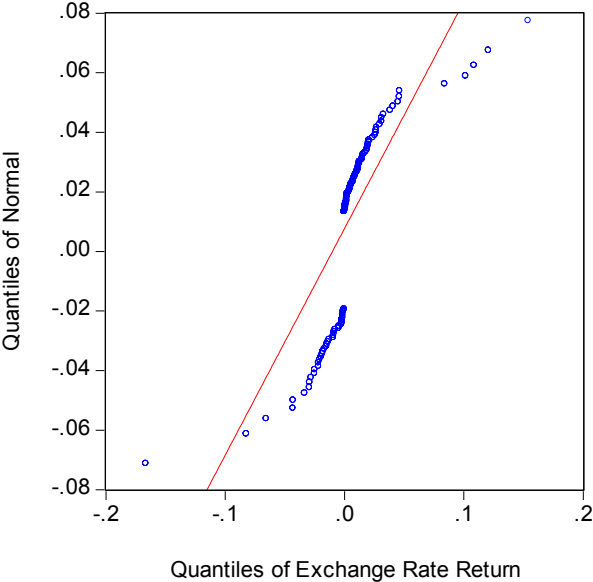


Figure 3: Quantile-Quantile Plot of Exchange Rate Return Series Based for the Managed Floating Rate Regime (September 1986 – December 2007)

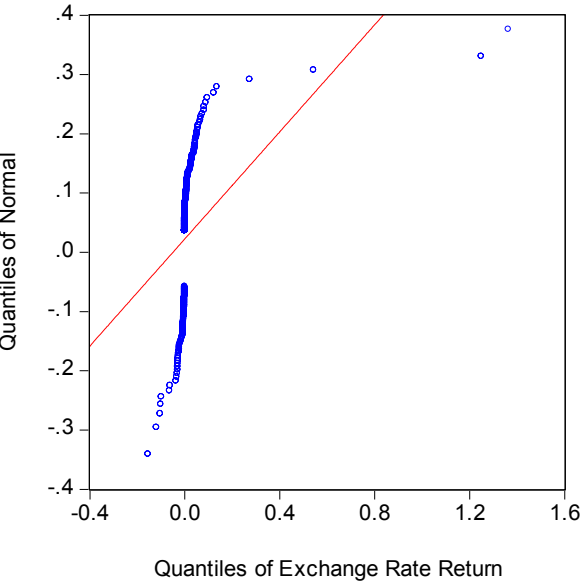


Table 2: Autocorrelation of the Power transformed Return series using Powers of 0.25, 0.5 and 0.75

	Full Sample	Fixed rate regime	Managed Float
Ljung-Box Q^{0.25} Statistics			
Q ^{0.25} (6)	377.980 (0.000)*	153.040 (0.000)*	196.680 (0.000)*
Q ^{0.25} (12)	635.100 (0.000)*	228.040 (0.000)*	341.650 (0.000)*
Q ^{0.25} (20)	859.980 (0.000)*	294.890 (0.000)*	460.770 (0.000)*
Ljung-Box Q^{0.5} Statistics			
Q ^{0.5} (6)	104.890 (0.000)*	92.210 (0.000)*	32.219 (0.000)*
Q ^{0.5} (12)	165.240 (0.000)*	126.140 (0.000)*	53.307 (0.000)*
Q ^{0.5} (20)	202.040 (0.000)*	160.290 (0.000)*	63.425 (0.000)*
Ljung-Box Q^{0.75} Statistics			
Q ^{0.75} (6)	17.670 (0.007)*	55.215 (0.000)*	3.932 (0.686)
Q ^{0.75} (12)	25.461 (0.013)*	67.201 (0.000)*	6.189 (0.906)
Q ^{0.75} (20)	26.991 (0.036)*	78.754 (0.000)*	6.397 (0.998)

Notes: p values are in parentheses. * indicates significance at the 5% level

Table 3: Unit Root Test of the Exchange rate Return Series over the period, January 1970-December 2007

	Augmented Dickey-Fuller test				Phillips-Perron test			
	Statistic	Critical Values (% level)			Statistic	Critical Values (% level)		
		1%	5%	10%		1%	5%	10%
Full Sample	-21.145	-2.570	-1.942	-1.616	-21.147	-2.570	-1.942	-1.616
Fixed rate regime	-7.588	-2.577	-1.942	-1.616	-16.001	-2.577	-1.942	-1.616
Managed Float	-15.790	-2.574	-1.942	-1.616	-15.789	-2.574	-1.942	-1.616

Notes: The appropriate lags are automatically selected employing Akaike information Criterion

3.2 Models Used In This Study

This study will attempt to model the volatility of monthly exchange rates return. The mean equation that will be used in this study is given as:

$$r_t = c + \varepsilon_t \quad \varepsilon_t / \phi_{t-1} \sim t(0, \sigma_t^2, \nu_t) \quad (10)$$

where ν_t is the degree of freedom

On September 29, 1986, the foreign exchange market was deregulated in Nigeria paving the way for the introduction of market determined exchange rates (managed floating exchange rate). To account for the introduction of managed floating exchange rate system, this paper introduced a dummy variable which is set equal to 0 for the period before the introduction of managed floating exchange rate and 1 thereafter. Thus, for the full sample, Equation (9) is adjusted as:

$$r_t = c + \text{dum}1 + \varepsilon_t \quad \varepsilon_t / \phi_{t-1} \sim t(0, \sigma_t^2, \nu_t) \quad (11)$$

For the two sub-periods, Fixed/Pegged exchange rate regime and Floating exchange rate regime, Equation (9) will still be used as the mean equation.

The volatility models used in this study include:

$$\text{GARCH}(1, 1): \quad \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (12)$$

$$\text{IGARCH}(1, 1): \quad \sigma_t^2 = \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad \text{with } \alpha + \beta = 1 \quad (13)$$

$$\text{TS-GARCH}(1,1): \quad \sigma_t = \omega + \alpha |\varepsilon_{t-1}| + \beta \sigma_{t-1} \quad (14)$$

$$\text{GJR-GARCH}(1,1): \quad \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^- \mathbf{I}_{t-1}^- \quad (15)$$

$$\text{EGARCH}(1,1): \quad \log(\sigma_t^2) = \omega + \alpha \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right| + \beta \log(\sigma_{t-1}^2) + \gamma \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| \quad (16)$$

$$\text{APARCH}(1,1) \quad : \quad \sigma_t^\delta = \omega + \alpha (|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1})^\delta + \beta \sigma_{t-1}^\delta \quad (17)$$

The volatility parameters to be estimated include ω , α , β , γ and δ . As the exchange rate return series shows a strong departure from normality, all the models will be estimated with Student t as the conditional distribution for errors. The estimation will be done in such a way as to achieve convergence. Where necessary, the Student t will be chosen with a fixed degree of freedom so as to achieve convergence. All the models will be evaluated using the Akaike, Schwarz and Hannan-Quinn criteria.

4. The Results

The results of estimating Equations(10) to (17) are presented in Tables 4, 5 and 6. Table 1 shows the result for the full sample over the period January 1970-December 2007. Table 5 shows the result for the first sub-period (Fixed rate regime) over the

period, January 1970-August 1986 while Table 6 shows the result for the second sub-period (Managed floating rate regime) over the period, September 2006 - December 2007. Table 4 shows that, in the mean equation, the coefficient of the dummy variable (proxy for change from fixed rate regime to managed float) is not significant at the 5% level in the GARCH, EGARCH and APARCH models but significant in the GJR-GARCH, IGARCH and TS-GARCH models. This shows that the impact of change from the fixed rate regime to managed float depends on the choice of model. The change to managed float regime is important in only the GJR-GARCH, IGARCH and TS-GARCH models.

The variance equation of Table 4 shows that α , the coefficient is not statistically significant in the GARCH and EGARCH models but significant at the 5% level in GJR-GARCH, APARCH, IGARCH and TS-GARCH models. This appears to show the presence of volatility clustering in GJR-GARCH, APARCH, IGARCH and TS-GARCH models. Conditional volatility for these models tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller). A breakdown of the results shows that the statistical coefficients of α in Table 5 (Fixed rate period) are similar to those in Table 4 for all models. However, in Table 6 (managed floating rate regime), the coefficients of α are statistically significant at the 5% level in the APARCH, IGARCH and TS-GARCH models. This shows that the shift from fixed rate regime to managed float affected the statistical significance of α in the GJR-GARCH model.

Table 4 shows that the coefficients of β (a determinant of the degree of persistence) is statistically significant in all models. This result for the coefficient of β is the same for the two sub periods (Fixed rate and managed float regime) as shown in Tables 5 and 6 showing the shift from fixed rate regime to managed float did not affect the statistical significance of β . The sum of α and β in the GARCH model in Tables 4, 5 and 6 exceeds 1. This appears to show that the shocks to volatility are very high and will remain forever as the variances are not stationary under the GARCH model. However, in view of the insignificance of α in the GARCH model, the result is inconclusive. In the GJR-GARCH and APARCH models of Tables 4, 5 and 6 , $\alpha + \beta + (\gamma/2)$ exceeds 1. This also appears to show that the shocks to volatility are very high and the variances are not stationary under the GJR-GARCH and APARCH models. However, the EGARCH models of Tables 4, 5 and 6 have their β s below 1 showing persistent volatility in the EGARCH model. IGARCH model is already modeled to have $\alpha + \beta$ equal to 1. The results in Tables 4, 5 and 6 show that IGARCH model fit our exchange rate return data as both the α and β are statistically significant at the 5% level. The IGARCH model also shows that variances are not stationary and persistence of volatility will remain forever. The sum of α and β in the TS-GARCH model in Tables 4, 5 and 6 exceeds 1. This appears to show that the shocks to volatility are very high and will remain forever as the variances are not stationary under the TS-GARCH model. The volatility persistence for the GARCH, GJR-GARCH and E-GARCH models in Table 5 are higher than those of Table 6 indicating that volatility persistence is higher in the Managed floating rate regime than Fixed rate regime. However, in view of the insignificance of α in the GARCH model and GJR-model of Table 6, this result is inconclusive. The APARCH and TS-GARCH models have their volatility persistence higher in Table 5 compared to Table

6 showing that a shift from fixed rate regime to reduces volatility persistence. However, in sum, the Nigerian Foreign exchange market is characterized by high volatility persistence.

Table 4: Parameter Estimates of GARCH Models For the Full Sample, January 1970 - December 2007

	GARCH	GJR-GARCH	EGARCH	APARCH	IGARCH	TS-GARCH
Mean Equation						
c	0.000 (0.000)*	0.000 (0.000)*	0.000 (0.001)	0.000 (0.000)	0.000 (0.000)*	0.000 (0.000)*
dum1	0.000 (0.000)	0.000 (0.000)*	0.000 (0.001)	0.000 (0.000)	0.000 (0.000)*	0.000 (0.000)*
Variance Equation						
ω	0.000 (0.000)	0.000 (0.000)	-2.047 (0.572)*	0.007 (0.005)		0.000 (0.000)
α	8.363 (5.480)	3.892 (0.482)*	1.090 (1.128)	1.477 (0.490)*	0.288 (0.005)*	2.117 (0.649)*
β	0.129 (0.007)*	0.126 (0.008)*	0.720 (0.028)*	0.562 (0.020)*	0.712 (0.005)*	0.442 (0.012)*
γ		-2.547 (0.497)*	-0.734 (0.769)	-0.201 (0.074)*		
δ				0.440 (0.035)*		
ν	2.130 (0.095)*	3.000	2.027	2.008 (0.012)*	2.170 (0.017)*	2.079 (0.051)*
LL	2170.541	1908.984	1291.976	2025.203	1642.133	2050.623
Persistence	8.492	2.745	0.720	1.939	1.000	2.559
AIC	-9.514	-8.365	-5.648	-8.867	-7.201	-8.987
SC	-9.460	-8.310	-5.585	-8.794	-7.164	-8.933
HQC	-9.493	-8.343	-5.623	-8.838	-7.186	-8.966
N	455	455	455	455	455	455

Notes: Standard errors are in parentheses. * indicates significant at the 5% level.

LL, AIC, SC, HQC and N are the maximum log-likelihood, Akaike information Criterion, Schwarz Criterion, Hannan-Quinn criterion and Number of observations respectively

Table 5: Parameter Estimates of GARCH Models For the Fixed Exchange Rate Regime, January 1970 - August 1986

	GARCH	GJR-GARCH	EGARCH	APARCH	IGARCH	TS-GARCH
Mean Equation						
Constant	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Variance Equation						
ω	0.000 (0.000)	0.000 (0.000)	-4.913 (1.065)*	0.000 (0.000)		0.000 (0.000)
α	2.255 (2.191)	1.524 (0.211)*	2.006 (2.178)	2.008 (0.507)*	0.984 (0.007)*	2.759 (0.891)*
β	0.401 (0.010)*	0.354 (0.008)*	0.507 (0.036)*	0.603 (0.027)*	0.016 (0.007)*	0.341 (0.010)*
γ		-0.770 (0.235)*	1.632 (1.773)	-0.466 (0.072)*		
δ				0.636 (0.054)*		
ν	2.106 (0.113)*	7.000	2.056 (0.129)*	2.009	2.102 (0.024)*	2.057 (0.038)*
L	837.244	684.366	657.542	1012.160	659.801	1013.980
Persistence	2.655	1.493	0.507	2.378	1.000	3.099
AIC	-8.364	-6.828	-6.548	-10.102	-6.601	-10.141
SC	-8.282	-6.745	-6.449	-9.986	-6.551	-10.058
HQC	-8.331	-6.794	-6.508	-10.055	-6.581	-10.107
N	199	199	199	199	199	199

Notes: Standard errors are in parentheses. * indicates significant at the 5% level.

LL, AIC, SC, HQC and N are the maximum log-likelihood, Akaike information Criterion, Schwarz Criterion, Hannan-Quinn criterion and Number of observations respectively

Table 6: Parameter Estimates of GARCH Models For the Managed Floating Rate Regime, September 1986 - December 2007

	GARCH	GJR-GARCH	EGARCH	APARCH	IGARCH	TS-GARCH
Mean Equation						
Constant	0.000 (0.000)	0.000 (0.000)	0.000 (0.001)	0.000 (0.000)	0.000 (0.000)*	0.000 (0.000)
Variance Equation						
ω	0.000 (0.000)	0.000 (0.000)	-3.396 (0.913)*	0.030 (0.012)*		0.000 (0.000)
α	6.940 (5.498)	7.865 (11.643)	1.353 (1.121)	1.012 (0.186)*	0.645 (0.015)*	1.710 (0.708)*
β	0.096 (0.010)*	0.043 (0.006)*	0.534 (0.049)*	0.447 (0.027)*	0.355 (0.015)*	0.448 (0.015)*
γ		1.613 (3.532)	-0.945 (0.803)	-0.303 (0.102)*		
δ				0.248 (0.035)*		
ν	2.186 (0.176)*	2.074 (0.119)*	2.039 (0.067)*	2.010 (0.011)*	2.584 (0.081)*	2.124 (0.115)*
L	1249.661	1243.049	678.881	1193.575	1076.070	1251.168
Persistence	7.035	8.714	0.534	1.307	1.000	2.158
AIC	-9.724	-9.664	-5.257	-9.270	-8.383	-9.736
SC	-9.655	-9.581	-5.174	-9.173	-8.342	-9.666
HQC	-9.696	-9.631	-5.223	-9.231	-8.367	-9.708
N	256	256	256	256	256	256

Notes: Standard errors are in parentheses. * indicates significant at the 5% level.

LL, AIC, SC, HQC and N are the maximum log-likelihood, Akaike information Criterion, Schwarz Criterion, Hannan-Quinn criterion and Number of observations respectively

Table 4, 5 and 6 show that the coefficients of γ , the asymmetry and leverage effects, are negative and statistically significant at the 5% level in the GJR-GARCH and APARCH models but negative and insignificant in the EGARCH model. However, leverage effect will only exist if $\gamma > 0$ in the GJR-GARCH and APARCH models and $\gamma < 0$ in the EGARCH. In view of the statistical insignificance of γ in the EGARCH model and negative values of γ in the GJR-GARCH and APARCH models, the hypothesis of leverage effect is rejected for all models but asymmetry effect is accepted for the GJR-GARCH and APARCH models. The results of asymmetry and leverage effects are the same for the fixed rate and managed float regimes. The estimated coefficients of the degree of freedom, ν are significant at the 5-percent level in all models of Tables 4, 5 and 6 where standard errors are stated, implying the appropriateness of student t distribution. Standard errors were not estimated for models estimated under the distributional assumption of student t with fixed parameter.

Diagnostic Checks

Tables 7, 8 and 9 shows the results of the diagnostic checks on the estimated models in Tables 4, 5 and 6. Table 7 shows the results of the estimated checks on the full sample. Table 8 shows the result of the diagnostic check on the fixed rate regime and Table 9 shows the result of the diagnostic check for the managed floating rate regime. Tables 7, 8 and 9 show that the Ljung-Box Q-test statistics of the standardized residuals for the remaining serial correlation in the mean equation shows that autocorrelation of standardized residuals are statistically insignificant at the 5% level for all lags and models confirming the absence of serial correlation in the standardized residuals. This shows that the mean are well specified in all models in Tables 4, 5 and 6. The Ljung-Box Q²-statistics of the squared standardized residuals in Tables 7, 8 and 9 are all insignificant at the 5% level for all lags and models confirming the absence of ARCH in the variance equation. The ARCH-LM test statistics in Tables 7,8 and 9 for all models further showed that the standardized residuals did not exhibit additional ARCH effect. This shows that the variance equations are well specified in all models of Tables 4, 5 and 6. The Jarque-Bera statistics still shows that the standardized residuals are not normally distributed. In sum, all the models are adequate for forecasting purposes.

Table 7: Autocorrelation of Standardized Residuals, Autocorrelation of Squared Standardized Residuals and ARCH LM test of Order 4 for the Full Sample

	Ljung-Box Q-statistics			Ljung-Box Q ² Statistics			ARCH LM		JB
	Q(6)	Q(12)	Q(20)	Q ² (6)	Q ² (12)	Q ² (20)	F	N*R ²	
GARCH	0.010 (1.000)	0.021 (1.000)	0.036 (1.000)	0.014 (1.000)	0.028 (1.000)	0.048 (1.000)	0.002 (1.000)	0.009 (1.000)	3834332 (0.000)
GJR-GARCH	0.014 (1.000)	0.028 (1.000)	0.047 (1.000)	0.014 (1.000)	0.028 (1.000)	0.048 (1.000)	0.002 (1.000)	0.009 (1.000)	3873371 (0.000)
EGARCH	0.075 (1.000)	0.186 (1.000)	0.428 (1.000)	0.029 (1.000)	0.060 (1.000)	0.104 (1.000)	0.004 (1.000)	0.018 (1.000)	1640936 (0.000)
APARCH	0.009 (1.000)	0.019 (1.000)	0.032 (1.000)	0.014 (1.000)	0.028 (1.000)	0.048 (1.000)	0.002 (1.000)	0.009 (1.000)	3806252 (0.000)
IGARCH	0.014 (1.000)	0.028 (1.000)	0.047 (1.000)	0.014 (1.000)	0.028 (1.000)	0.048 (1.000)	0.002 (1.000)	0.009 (1.000)	3873367 (0.000)
TS-GARCH	0.013 (1.000)	0.026 (1.000)	0.045 (1.000)	0.014 (1.000)	0.028 (1.000)	0.048 (1.000)	0.002 (1.000)	0.009 (1.000)	3872038 (0.000)

Note: p values are in parentheses

Table 8: Autocorrelation of Standardized Residuals, Autocorrelation of Squared Standardized Residuals and ARCH LM test of Order 4 for the Fixed Exchange Rate Regime

	Ljung-Box Q-statistics			Ljung-Box Q ² Statistics			ARCH LM		JB
	Q(6)	Q(12)	Q(20)	Q ² (6)	Q ² (12)	Q ² (20)	F	N*R ²	
GARCH	0.127 (1.000)	0.265 (1.000)	0.453 (1.000)	0.115 (1.000)	0.242 (1.000)	0.414 (1.000)	0.019 (0.999)	0.076 (0.999)	87984.850 0.000
GJR-GARCH	0.128 (1.000)	0.267 (1.000)	0.453 (1.000)	0.119 (1.000)	0.249 (1.000)	0.421 (1.000)	0.019 (0.999)	0.078 (0.999)	85239.960 (0.000)

EGARCH	1.029 (0.985)	2.576 (0.998)	3.709 (1.000)	0.396 (0.999)	0.744 (1.000)	1.272 (1.000)	0.067 (0.992)	0.273 (0.992)	17450.860 (0.000)
APARCH	0.128 (1.000)	0.269 (1.000)	0.456 (1.000)	0.121 (1.000)	0.253 (1.000)	0.429 (1.000)	0.019 (0.999)	0.080 (0.999)	83828.350 (0.000)
IGARCH	0.309 (0.999)	0.426 (1.000)	0.441 (1.000)	0.044 (1.000)	0.094 (1.000)	0.160 (1.000)	0.007 (1.000)	0.028 (1.000)	81994.890 (0.000)
TS-GARCH	0.127 (1.000)	0.266 (1.000)	0.450 (1.000)	0.119 (1.000)	0.249 (1.000)	0.420 (1.000)	0.019 (0.999)	0.078 (0.999)	85451.490 (0.000)

Note: p values are in parentheses

Table 9: Autocorrelation of Standardized Residuals, Autocorrelation of Squared Standardized Residuals and ARCH LM test of Order 4 for the Managed Floating Rate Regime

	Ljung-Box Q-statistics			Ljung-Box Q ² Statistics			ARCH LM		
	Q(6)	Q(12)	Q(20)	Q ² (6)	Q ² (12)	Q ² (20)	F	N*R ²	JB
GARCH	0.025 (1.000)	0.051 (1.000)	0.090 (1.000)	0.025 (1.000)	0.051 (1.000)	0.090 (1.000)	0.004 (1.000)	0.016 (1.000)	682827 (0.000)
GJR-GARCH	0.025 (1.000)	0.052 (1.000)	0.090 (1.000)	0.025 (1.000)	0.051 (1.000)	0.090 (1.000)	0.004 (1.000)	0.016 (1.000)	682827 (0.000)
EGARCH	0.144 (1.000)	0.356 (1.000)	0.653 (1.000)	0.034 (1.000)	0.070 (1.000)	0.121 (1.000)	0.005 (1.000)	0.022 (1.000)	471319 (0.000)
APARCH	0.025 (1.000)	0.057 (1.000)	0.095 (1.000)	0.025 (1.000)	0.051 (1.000)	0.090 (1.000)	0.004 (1.000)	0.016 (1.000)	682811 (0.000)
IGARCH	0.025 (1.000)	0.051 (1.000)	0.090 (1.000)	0.025 (1.000)	0.051 (1.000)	0.090 (1.000)	0.004 (1.000)	0.016 (1.000)	682827 (0.000)
TS-GARCH	0.025 (1.000)	0.051 (1.000)	0.090 (1.000)	0.025 (1.000)	0.051 (1.000)	0.090 (1.000)	0.004 (1.000)	0.016 (1.000)	682827 (0.000)

Note: p values are in parentheses

Model Evaluation

Table 10 shows the ranking of the models in terms of the of maximum log-likelihood, lowest Akaike information, Schwarz and Hannan-Quinn criteria The best model for the full sample in terms of maximum log-likelihood is the GARCH model. The model also has the lowest Akaike information, Schwarz and Hannan-Quinn criteria. However, the next best model is TS-GARCH followed by the APARCH model. The GARCH model marginally outperforms the TS-GARCH followed by the APARCH models in terms maximum log-likelihood and Akaike information, Schwarz and Hannan-Quinn criteria. In view of the statistically insignificance of α in the GARCH model, the TS-GARCH and APARCH models might be preferable for the full sample. For the sub-periods (Fixed rate regime and managed float), the TS-GARCH model outperforms other models.

Table 10: Ranking of GARCH Models in Order of maximum log-likelihood, Akaike information Criterion, Schwarz Criterion, Hannan-Quinn criterion

Rank	Full Sample	Fixed Rate Regime	Managed Float
1 st	GARCH	TS-GARCH	TS-GARCH

2 nd	TS-GARCH	APARCH	GARCH
3 rd	APARCH	GARCH	GJR-GARCH
4 th	GJR-GARCH	GJR-GARCH	APARCH
5 th	IGARCH	IGARCH	IGARCH
6 th	EGARCH	EGARCH	EGARCH

5. Conclusion

This paper investigated the volatility of Naira/Dollar exchange rates using GARCH (1,1), GJR-GARCH(1,1), EGARCH(1,1), APARCH(1,1), IGARCH(1,1) and TS-GARCH(1,1) models. The TS-GARCH and APARCH models are the best models as they have all the parameters of the variance equations being significant. However, all the models except EGARCH model, show that there is high volatility persistence in Nigerian foreign exchange market with all variances being non-stationary. The APARCH model and GJR-GARCH model for the managed floating rate regime show the existence of statistically significant asymmetry effect. All the asymmetry models rejected the hypothesis of leverage effect. The high volatility persistence in the fixed exchange rate period could have been due to the import dependent nature of the Nigerian economy. The inadequate supply of foreign exchange by the Central Bank of Nigeria, the activities of foreign exchange dealers and parallel markets during could have contributed to the high volatility persistence during the fixed exchange rate period. In the managed floating rate regime, even though there is still high volatility persistence, the intervention of the Central Bank of Nigeria in the foreign exchange market could have moderated the volatility persistence during this period as shown in the APARCH and TS-GARCH models. Further research work needs to be done using other volatility models and higher frequency data. Researchers might also want to examine the impact of Central Bank intervention on exchange rate volatility.

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