A variable neighborhood search algorithm for the heterogeneous fleet vehicle routing problem for crossdocking in the supply chain

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Crossdocking is a relatively new logistics strategy which can reduce inventory holding, order picking and transportation costs while speed up the physical flow in the supply chain. The vehicle routing decisions also play an important role in the efficiency of the distribution system. This article addresses the integration problem of crossdocking into the heterogeneous fleet vehicle routing problem to introduce heterogeneous fleet vehicle routing problem with crossdocking. In the considered problem, a set of heterogeneous vehicles transport products from suppliers to customers via a crossdock. The problem was formulated as an integer programming model to determine the best vehicle routes and the optimal number of different types of vehicles minimizing the total operational and transportation cost. Since the considered problem is NP-hard, a heuristic algorithm based on a variable neighborhood search is suggested. Numerical experiments show that the proposed algorithm can find high quality solutions within an acceptable computation time.

1. Introduction

Being competitive in nowadays fast changing business environment, forces organizations to search for more profitable approaches to satisfy customer demands, reduce costs and increasing their responsibility. In supply chain management three main solutions of logistics performance enhancement are distribution network optimization, shipment consolidation, and crossdocking (Brockmann 1999).

Crossdocking is one of the main distribution strategies in which items are distributed continuously from suppliers through warehouses to customers. However the Distribution centers rarely hold the items for more than 10 to 15 hours. Using this strategy has a considerable impact on the reduction of material handling cost, customer orders turnaround times and storage space

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requirements. One of the key factors of success in crossdocking management is to reduce or eliminate the storage period. The shorter storage time requires smaller storage space obviously (Apte & Viswanathan 2000).

Two major points of crossdocking strategy are simultaneous arrival and consolidation (Lee at al. 2006). First all vehicles from suppliers must arrive at the crossdock simultaneously to avoid undesirable waits. So it is better to synchronize all pickups to reduce waiting times. The process of combination and classification of products according to income orders and their destinations is consolidation. If these two processes can be carried out successfully in a supply chain’s physical flow, all products can be moved from suppliers to customers without any interruptions (Lee at al. 2006).

On the other hand the vehicle routing plays an important role in the logistics planning. The classic vehicle routing problem (VRP) includes the service to customers with known demands by a fleet of vehicles from a distribution center. The objective of this problem is to minimize the transportation cost and the operational cost of vehicles. Many different versions of VRP have been considered to provide a variety range of real world conditions. Heterogeneous fleet vehicle routing problem (HVRP) which represents the generalized form of the VRP involves various type of vehicles with different predefined capacity and cost.

In this study we integrate crossdocking into the HVRP to introduce heterogeneous fleet vehicle routing problem with crossdocking (HVRPCD). Here a set of heterogeneous vehicles transport products from suppliers to customers via a crossdock. The system uses post-distribution crossdocking strategy. Therefore inbound freights are accumulated and consolidated in minimum possible time before dispatching to customers. The vehicles can serve more than one supplier in a pickup route or several customers in a delivery route regarding to their capacity; but these two routes are never inter-mixed.

The problem was formulated as an integer programming model to determine the best vehicle routes and the optimal number of different types of vehicles minimizing the total operational cost. Since the considered problem is NP-hard, in this study a heuristic algorithm based on a variable neighborhood search (VNS) is suggested. Numerical experiments show that the proposed algorithm can find high quality solutions within an acceptable computation time.

The organization of this article is as follows: Section 2 provides a brief review of vehicle routing and distribution planning in crossdocking studies. In section 3 the definition of the proposed problem and its mathematical model will be expressed. The proposed solution algorithm based on VNS will be illustrated in section 4. Section 5 reports the computational results of the proposed algorithm on some random test problems. Finally section 6 ends the paper with concluding notes.

2. Literature Review

Crossdocking Strategy is almost a new logistics concept and there are not much academic literatures about it. Recently Agustina et al. (2010) reviewed the mathematical models in crossdocking planning and offered a classification based
on considered decision levels. Scheduling, duck door assignment, transhipment and vehicle routing are remarked as operational decision, while layout planning and network design problems are classified in tactical and strategic level respectively. It must be noted that most efforts in crossdocking studies have been focused in the scheduling problem of inbound and outbound trucks. As these studies are not related to our work, readers are referred to Boysen and Fliedner (2010) for more details. In the following we will shortly describe the past related researches to the distribution planning and vehicle routing scheduling in crossdocking networks.

Sung and Song (2003) investigated an integrated service network design problem that integrates crossdocks location problem and vehicle allocation problem for associated services from origins to crossdocks and from crossdocks to destinations. They formulated the problem based on paths and solved by using a tabu search algorithm. Sung & Yang, (2006) also presented a set-partitioning-based formulation and a branch-and-price solution method for this problem. Gumus and Bookbinder (2004) determined both best portion of each transportation policy and optimal locations of crossdocks within a medium sized distribution network. However they used commercial softwares and did not provide any heuristic solution procedure. Lim et al., (2005) addressed just-in-time objectives studying the transhipment problem in a crossdocking network which include shipping and delivery time windows and transportation constraints. They provided optimality conditions for polynomially solvable cases and complexity proofs for other discussed problems. Chen et al. (2006) studied the distribution planning problem for a crossdocking network considered delivery and pick-up time windows, warehouse capacities and inventory handling costs. They proved the NP hardness of the problem and presented simulated annealing and tabu search heuristics as solution methods. Musa et al. (2010) dealt with the transportation problem of a distribution network where the loads can be transferred directly from suppliers to customers or through crossdocking facilities. The objective was to create a truck operating plan which minimizes the total transportation cost. They used an ant colony optimization algorithm to solve the problem.

The integration of crossdocking and VRP was first introduced by Lee at al. (2006). They considered a network of multiple suppliers, one crossdock and multiple customers. A mathematical formulation of the problem and a tabu search algorithm were developed to find best vehicle routing schedule. Liao et al. (2010) continued this work and proposed a new tabu search algorithm. Wen et al., (2009) treated the VRP in a distribution network where a set of homogeneous vehicles are used to transport products from the suppliers to customers through a crossdock. The problem was formulated as a mixed integer programming which minimized the total travelled distance while respecting both orders time window constraints and a time horizon for the whole transportation operation. They proposed a tabu search heuristic within an adaptive memory procedure to solve the problem.
3. Problem Definition

We consider a distribution network made of a set $P$ of suppliers, a set $D$ of customers and one crossdock. The supplies and demands are taken as pickups and deliveries respectively. The distribution system employs a post-distribution strategy. Therefore inbound freights must be accumulated and consolidated in minimum possible time before dispatching to customers. A fleet of heterogeneous vehicles with known capacities and different transportation and operational costs will be scheduled to meet pickup and delivery nodes. The number of available vehicles of each type is assumed to be limited. We consider a set of feasible routes to serve supplies and demand. The vehicles can serve multiple nodes en route regarding to their capacity. The number of routes is assumed to be equal to the total number of pickup and delivery nodes. This is because the worse case is occurred when each node is served separately. But in other cases some of the routes are not required to be planned. The pickup and delivery processes are assumed to take place separately. The treated network is shown in Fig. 1. The problem also has some timing considerations. To eliminate unnecessary waiting at the crossdock, all pickup routes must arrive at the crossdock simultaneously. Further the planning horizon is finite and all processes should be accomplished during this period.

![Fig. 1. The considered crossdocking network](image)

The objective of this problem is to determine the number of required vehicles of each type and the best schedule and route of each vehicle to minimize the total transportation and operational costs. With these definitions, the following integer programming model is formulated to describe the considered problem.

### 3.1 Sets:

- $P$: Set of pickup nodes
- $D$: Set of delivery nodes
- $N$: Set of all nodes including crossdock; $N = P \cup D \cup \{0\}$
- $K$: Set of Vehicle types
- $R$: Set of all feasible routes
3.2 Parameters:

- $TC_{i,j}^k$: Transportation cost from node $i$ to node $j$ by vehicle type $k$
- $OC_k$: Operational cost of the vehicle type $k$
- $V_k$: Number of type $k$ vehicles available at the crossdock
- $P_i$: The quantity loaded in the pickup node $i$
- $D_i$: The quantity unloaded in the Delivery node $i$
- $CAP_k$: Capacity of vehicle type $k$
- $T_{i,j}$: Moving time from node $i$ to node $j$
- $T^s_i$: Stopping time required at node $i$
- $T$: Planning horizon

3.3 Variables:

- $x_{r,i,j}^k$: $1$ if route $r$ includes the trip from node $i$ to node $j$ by vehicle type $k$, $0$ otherwise
- $z_k$: Number of employed vehicle type $k$
- $t^D_r$: Departure time of route $r$ from the crossdock in the pickup process
- $t^A_{r,i}$: Arrival time of route $r$ to pickup node $i$
- $t^A_r$: Arrival time of route $r$ to the crossdock in the pickup process

3.4 Model:

Minimize
\[ \sum_{k \in K} \sum_{r \in R} \sum_{i \in N} \sum_{j \in N} TC_{i,j}^k x_{r,i,j}^k + \sum_{k \in K} OC_k z_k \] (1)
Subject to:
\[ \sum_{k \in K} \sum_{r \in R} \sum_{i \in N} x_{r,i,j}^k = 1 \quad \forall j \in N - \{0\} \] (2)
\[ \sum_{k \in K} \sum_{r \in R} \sum_{j \in N} x_{r,i,j}^k = 1 \quad \forall i \in N - \{0\} \] (3)
\[ \sum_{i \in N} x_{r,i,l}^k = \sum_{j \in N} x_{r,j,l}^k \quad \forall r \in R, \forall l \in N & \forall k \in K \] (4)
\[ \sum_{k \in K} \sum_{j \in N} x_{r,0,j}^k \leq 1 \quad \forall r \in R \] (5)
\[ \sum_{r \in R} \sum_{j \in N} x_{r,0,j}^k = z_k \quad \forall k \in K \] (6)
\[ z_k \leq V_k \quad \forall k \in K \] (7)
\[ \sum_{i \in N - \{0\}} (P_i + D_i) \sum_{j \in N} x_{r,i,j}^k \leq CAP_k \quad \forall r \in R & \forall k \in K \] (8)
\[ \sum_{k \in K} \sum_{r \in R} \sum_{e \in P} x_{r,i,j}^k = 0 \quad \forall i \in P \] (9)
\[ \sum_{k \in K} \sum_{r \in R} \sum_{e \in D} x_{r,i,j}^k = 0 \quad \forall i \in D \] (10)
\[ \sum_{k \in K} \sum_{r \in R} x_{r,i,l}^k = 0 \quad \forall i \in N \] (11)
\[ \sum_{k \in K} \sum_{r \in R} (x_{r,i,j}^k + x_{r,j,i}^k) \leq 1 \quad \forall i,j \in N - \{0\} \] (12)
\[ t^A_{r,j} = (t^D_r + T_{0,j}) \sum_{k \in K} x_{r,0,j}^k + \sum_{l \in P} (t^A_{r,l} + T^s_l + T_{l,j}) \sum_{k \in K} x_{r,l,j}^k \] (13)
\[ \forall r \in R & \forall j \in P \]
Objective function (1) minimizes the total transportation cost and total fixed operational cost of vehicles. Equations (2) and (3) express that just one route has to arrive at and leave each node. The consecutive movements of vehicles en routes are guaranteed by constraint (4). Constraint (5) states that each route, if plans, uses one type of vehicle. Equation (6) determines the numbers of each type of vehicles. Constraint (7) ensures that the number of vehicles of type $k$ that leave the crossdock is available. Constraint (8) expresses that the quantity of loaded products on the pickup or delivery route cannot exceed the capacity of selected vehicle type. Equation (9) and (10) ensure that pickup and delivery processes are not intermixed. Constraint (11) and (12) prevent from loop and backward movement en routes respectively. The arrival times at pickup nodes and at crossdock are computed by equations (13) and (14) respectively. Constraint (15) guarantees the simultaneous arrival to crossdock for pickup routes. The arrival time of routes at crossdock and the total length of each route are restricted to the planning horizon, $T$, in constraints (16) and (17) respectively. Constraints (18)-(22) state the integrality restrictions on the decision variables.

4. The Methodology

The HVRP is an NP-hard combinatorial optimization, since it includes the classical VRP as a special case which is a generalization of the classical travelling salesman problem. Therefore in this article, we developed a heuristic algorithm based on VNS to solve the problem. VNS was first proposed by Mladenovic and Hansen (1997) for solving combinatorial and global optimization problems. This Meta-heuristic is mainly based on idea of a systematic change of neighborhoods within a local search method. During the search, the local optimum in each neighborhood structure is found. The process continues iteratively to hopefully achieve the global optimum at the end. More detailed studies on VNS can be found in Hansen and Mladenovic (1999, 2002). The proposed algorithm starts with a random initial solution. Then the algorithm will be run based on the below pseudo-code procedure.
1. Begin
2. Select a set of neighborhood structures \( N_k (k = 1,2, \ldots, k_{\text{max}}) \)
3. Generate an initial random solution \( X_{\text{best}} \)
4. Repeat the following steps
5. Set \( k \leftarrow 1; \)
6. Repeat the following Steps
7. \( X_{\text{curr}} \leftarrow X_{\text{best}}; \)
8. Generate \( X' \) randomly from the \( k \)th neighborhood structure, \( X' \in N_k (X_{\text{curr}}); \)
9. Apply a local search on \( N_k (X') \) to find best neighbor \( X''; \)
10. If \( f(X'') < f(X_{\text{curr}}) \) then
11. \( X_{\text{best}} \leftarrow X''; \)
12. Go to the step 5;
13. Else
14. \( k \leftarrow k + 1; \)
15. Until \( k = k_{\text{max}}; \)
16. Until \( \text{iteration} = \text{iteration}_{\text{max}} \)
17. End

We consider four neighborhood structures which include route change, order inversion, vehicle change, and node swap. These structures are briefly described in the following.

1. Route Change: In this structure the related route of a random node is changed randomly. The node may be add to the end of an exist route or planned to a new route.
2. Order inversion: Here the order of two randomly selected nodes in a route is reversed.
3. Vehicle change: Within this structure, the related vehicle of a random route is changed randomly.
4. Node Swap: Through this mechanism, two nodes from two different routes, but in same process, are selected randomly and both their routes and orders are swapped.

5. The findings:

In this section the results of some numerical experiments carried out to evaluate the performance of proposed heuristic algorithm is presented. We generate eight different instances randomly by varying the number of pickups, number of deliveries and number of different types of vehicles. In order to evaluate the performance of the algorithm, the optimal solutions of instances are required. We applied a time consumption enumeration algorithm to find the best of all feasible solutions. Then, with respect to the optimal solutions, the errors of the solutions obtained by the heuristic algorithm are computed. These experiments are summarized in table 1. Since the average error of total cost between the results of the proposed algorithm and optimal solutions for all tested instances is about 4
% and the maximum error is less than 9 %, it can be concluded that the proposed VNS algorithm works quite well to find good solutions efficiently.

<table>
<thead>
<tr>
<th>Instance #</th>
<th>Problem size</th>
<th>Proposed Algorithm</th>
<th>Optimal Solution</th>
<th>Error (%)</th>
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<tbody>
<tr>
<td>T1</td>
<td>4 × 6 × 2</td>
<td>7420</td>
<td>7420</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>5 × 10 × 3</td>
<td>9370</td>
<td>9260</td>
<td>1.19</td>
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<tr>
<td>T3</td>
<td>5 × 15 × 3</td>
<td>12160</td>
<td>11940</td>
<td>1.84</td>
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<tr>
<td>T4</td>
<td>5 × 20 × 4</td>
<td>14210</td>
<td>13880</td>
<td>2.38</td>
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<tr>
<td>T5</td>
<td>10 × 20 × 4</td>
<td>15820</td>
<td>15310</td>
<td>3.33</td>
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<tr>
<td>T6</td>
<td>10 × 30 × 5</td>
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<td>5.85</td>
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<tr>
<td>T7</td>
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<td>21580</td>
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<tr>
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</tr>
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</table>

6. Summary and Conclusions

Crossdocking is a relatively new logistics strategy that has a great potential to control the distribution costs and to maintain the level of customer service simultaneously. This article dealt with the HFVRP in a crossdocking network considering various capacity and costs of different vehicle types, restricted number of vehicles, simultaneous arrival of vehicles at the crossdock and time horizon constraint. The aim of this study is to determine the optimal number of required vehicles of each type and the best schedule and route of each vehicle minimizing total transportation and operational costs. The problem is formulated as an integer programming model and a heuristics solution procedure based on VNS is proposed. The experimental results show the superior performance of the proposed algorithm in comparison with optimal solutions. The average error between the results of the algorithm and optimal solutions is about 4 % and the maximum error is less than 9 % for all tested instances.

References


