Comparison of Performance of Optimal Portfolio Based on Various Risk estimations

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Most investors are willing to invest assets which generate higher return at lower risk. For the past decades, many methodologies introduced to measure and forecast the performance of portfolio and to manage risk. Many researcher have suggested various methodologies of performance measures in the investment decision making as an alternative to replace the Markowitz Mean-Variance Model or the Sharpe ratio. Biglova, Ortobelle, Rachev and Stoyanov(2004) compared 11 approaches to risk estimation for the optimal allocations among the risk free asset and nine risky assets in the German market. They report that the Sharpe ratio criterion under performs Minimax, MAD, and VaR criteria. In this paper, we try to find which portfolio criterion deduces best decision for 3 countries such as Korea, Japan and UK stock market. Industrial indices' rates of return are used as underlying asset's return in the sample period. We found that the performance of optimal portfolio derived from classic models such as sharpe ratio and markowitz is inferior to alternative models for each country's stock market in which the return on assets are not normally distributed in common.

Key words: Risk Estimation, Portfolio performance, Sharpe Ratio

1. Introduction

Most investors are willing to invest assets which generate higher return at lower risk. For the past decades, many methodologies introduced to measure and forecast the performance of portfolio and to manage risk. And William Sharpe(1996) invented the methodology "Reward-to-Variability" on the basis of Market equilibrium theory.

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This methodology had been famous as “Sharpe Ratio”, which assumes the distribution of rate of return is normal. But for the past decade, market equilibrium theory has been criticized because of unrealistic assumption on the return distribution. Practically, values of skewness and kurtosis of the return distribution are different from those of the normal distribution, so it may mislead investors. Thus, many researcher have suggested various methodologies of performance measures in the investment decision making as an alternative to replace the Markowitz Mean-Variance Model or the Sharpe ratio. Young(1998) proposed minimax optimal portfolio selection rule and Konno and Yamazaki(1991) suggested Mean-Absolute deviation portfolio optimization model. Favre and Galeano(2002) presented Mean-Modified Value-at-Risk Optimization model as a substitute. Yitzhaki(1982) and Shalt and Yitzhaki(1984,2005) suggested the Mean-Gini portfolio selection rule. Markowitz(1959), Fishburn(1977), Bawa(1977) suggested semivariance concept which considers downside risk. And, Sortino(2000) suggested upside potential ratio which considered the return that exceeded target return as rewards. These alternatives are valid theoretically, but we don’t know which criterion we should use for the derivation of the weights of the optimal portfolio. Bigova, Ortobelle, Rachev and Stoyanov(2004) compared 11 approaches to risk estimation for the optimal allocations among the risk free asset and nine risky assets in the German market. They report that the Sharpe ratio criterion under performs Minimax, MAD, and VaR criteria.

In this paper, we try to find which portfolio criterion deduces best decision for 3 countries such as Korea, Japan, and UK. Industrial indices’ rates of return are used as underlying asset’s return in the sample period. It is observed that distributions of indices’ returns do not have the normal distribution. We analyzed optimal portfolio for each country and found the performance of optimal portfolio derived from classic models such as sharpe ratio and markowitz is inferior to alternative models for each country’s stock market in which the return on assets are not normally distributed in common. In this study we presented several portfolio selection rules and compared their performances by analyzing actual stock market data. Sections 2 introduce optimal portfolio selection rules including sharpe ratio and markowitz which are most popular models. In section 3 we showed empirical comparisons. And section 4 is concluding remarks.

2. Portfolio Selection Rules

In this study, following symbols are used in order to compare different risk measures.

r_{jt} : rate of return of j - th asset at time t (\$1)

$\overline{r_j}$: average rate of return to asset j $(= 1/T \sum_{t=1}^{T} \mathrm{r_{jt}})$

w_j : weight of j - th asset

r_{pt} : portfolio of rate of return at time t $(= \sum_{t=1}^{N} \mathrm{w}_j r_{jt})$

$E[r_p]$: expected portfolio rate of return $(= \sum_{j=1}^{N} \mathrm{w}_j \overline{r_j})$

σ_p^2 : variance of portfolio rate of return

r_f : risk free rate

(1) Markowitz's Model

Markowitz proposed quantitative approach to portfolio analysis and uesd standard deviation as risk measure. Markowitz' theory is easy to understand so that is widely used until now.
Markowitz proposed the model which choose portfolio that minimize risk under the certain constraint to the average rate of return when choosing portfolio. That is, Markowitz invented what can minimize portfolio risk as long as meeting investor's minimum required rate of return.

(2) Sharpe Ratio

Sharpe's(1966) CAPM is what we extended to an equilibrium model from Markowitz's theory. In other words, like Markowitz portfolio selection theory, CAPM is one that explains how equilibrium price of capital assets (including stocks) are decided when the market is in a equilibrium with investors' behavior of the maximization of expected utility. Sharpe (1966, 1994) showed how investors expect and evaluate the performance of portfolio under equilibrium

$$\text{Sharpe Ratio} = \frac{\mathrm{E}[\mathrm{r_p}] - r_f}{\sigma_p}$$

$$\text{Max Sharpe Ratio} = \frac{\mathrm{E}[\mathrm{r_p}] - r_f}{\sigma_p}$$
$$s.t.$$
$$\sum_{j=1}^{n} w_j = 1$$
$$\sum_{j=1}^{N} w_j \overline{r_j} \geq G$$
$$w_j \geq 0,\ j = 1,\ldots,n$$

(3) Mini-Max Optimal Portfolio Selection Rule

Young(1998) proposed Mini-Max Model as an criterion for selecting optimal portfolio. This model minimizes maximum loss using past data. He showed that his model outperform Markowitz's Mean-Variance model in case the rate of return has non-Gaussian distribution of return.

$$\text{Max } \mathrm{M}_p$$
$$s.t.$$
$$\sum_{j=1}^{N} w_j r_{jt} - M_p \geq 0, t = 1,\ldots,T$$
$$\sum_{j=1}^{n} w_j = 1$$
$$\sum_{j=1}^{N} w_j \overline{r_j} \geq G$$
$$w_j \geq 0,\ j = 1,\ldots,n$$

(4) Mean Absolute Deviation

Konno and Yamazaki(1991) have created a method that uses MAD (Mean-Absolute Deviation) as measure of risk. This method was invented to cover shortcoming that are complex in business when optimizing large scale of portfolio in the Markowitz' model.

$$\min \ \mathrm{w(x)} = \mathrm{E}\left[\left|\sum_{j=1}^{N} r_j w_j - E[\sum_{j=1}^{N} r_j w_j]\right|\right]$$

$$s.t.$$

$$\sum_{j=1}^{n} w_j = 1$$

$$\sum_{j=1}^{N} w_j \overline{r_j} \geq G$$

$$w_j \geq 0,\ j = 1,\ldots,n$$

(5) Mean-Modified Value-at-Risk

Favre and Galeano(2002) adjusted the value-at-risk method by using an empirical VaR and an analytical VaR, which considered the skewness and the kurtosis. Therefore it can be used for non-normally distributed assets.

$$\mathit{Modified}\ \mathrm{VaR} = W\left[\mu - \left(z_c + \frac{1}{6}\left(z_c^2 - 1\right)S + \frac{1}{24}\left(z_c^3 - 3z_c\right)K - \frac{1}{36}\left(2z_c^3 - 5z_c\right)S^2\right)\sigma\right]$$

Z_c : critical value for probability $(1-\alpha)$

S : skewness

K : excess kurtosis

z_c has -2.33 in probability of 99%, -1.96 in probability of 95%. Linear Programming Model that minimizes modified-VaR is as follows.

$$\mathrm{Min} \ \ \mathrm{w(x)} = \mathrm{Modified\ VaR}$$

$$s.t.$$

$$\sum_{j=1}^{n} w_j = 1$$

$$\sum_{j=1}^{N} w_j \overline{r_j} \geq G$$

$$w_j \geq 0,\ j = 1,\ldots,N$$

(6) Gini Coefficient

Yitzhaki(1982), Shalt and Yitzhaki(1984, 2005) have suggested Mean-Gini portfolio selection throem that is consistent with a stochastic dominance rule. It is based on the expected value of the absolute difference between every pair of realization and can be calculated as below.

$$\Gamma_{\mathrm{p}}(\mathrm{v}) = -v * \mathrm{cov}\left\{R_p, [1 - F_p(R_p)]^{v-1}\right\}$$

The parameter v represents the investor's attitude toward risk. $F(R_p)$ is cumulative distribution function(CDF) of R_p. We use v=2 which indicates that investors have risk-averse.

$$\min\ \Gamma_{\mathrm{p}}(2) = -2 * \mathrm{cov}\left\{R_p, [1 - F_p(R_p)]^{2-1}\right\}$$

$$s.t.$$

$$\sum_{j=1}^{n} w_j = 1$$

$$\sum_{j=1}^{N} w_j \overline{r_j} \geq G$$

$$w_j \geq 0,\ j = 1, \ldots, n$$

(7) Target Semivariance and Target Absolute Semideviation

Markowitz(1959), Bawa(1977), Fishburn(1977) suggested semivariance. We use Target semivariance calculated by measuring downside risk of taret return.

$$\min\ T\mathrm{SV}(\text{target semivariance}) = 1/T\sum_{t=1}^{T} | TR - R_p |_{+}^{2}\ .$$

$$\sum_{j=1}^{n} w_j = 1$$

$$\sum_{j=1}^{N} w_j \overline{r_j} \geq G$$

$$w_j \geq 0,\ j = 1,...,n$$

TR is target return that must be earned at a minimum and $|TR\text{-}R_p|_{+}$ means that it has value if positive, otherwise 0. We also use Target absolute semideviation as the risk measure as below.

$$\min\ T\mathrm{A}SD(\text{target absolute semivariance}) = 1/T\sum_{t=1}^{T} | TR - R_p |_{+}\ .$$

$$\sum_{j=1}^{n} w_j = 1$$

$$\sum_{j=1}^{N} w_j \overline{r_j} \geq G$$

$$w_j \geq 0,\ j = 1,...,n$$

The only difference between two models is its power of objective function. These models are also related to Yong-Joo Lee, Kyung-Hee Jin(2000) in which the models are applied to Korean stock market.

(8) Sortino's Upside Potential Ratio

If the devations below target return are considered as risk which called downside risk, deviations above target return can be considered as reward. Sortino(2000) suggested the upside potential ratio which is similar to sharpe ratio.

$$\max \ Upside\ potential\ ratio = \frac{\int_{mar}^{+\infty} (R - mar)^1 f(R) dr}{\left[\int_{-\infty}^{mar} (R - mar)^2 f(R) dr \right]^{1/2}}$$

$$\sum_{j=1}^{n} w_j = 1$$

$$\sum_{j=1}^{N} w_j \overline{r_j} \geq G$$

$$w_j \geq 0, \ j = 1, \ldots, n$$

Where mar is minimal acceptable return which can be defined as TR.

3. Empirical Comparisons

In this study, we analyzed the optimal portfolio's performance for 3 countries. In order to compare optimal portfolio selection rule respectively, we used industrial indices of each country. We used 22 KOSPI industrial indices in the Korea stock market from Jan 5, 1987 to Jun 29, 2007. and 33 TOPIX industry indices in the Japanese stock market from Jan 4, 1985 to Jun 29, 2007 and 30 FTSE industry indices

<Table 1> Equally Weighted average daily return for each country

year	**KOSPI**	**TOPIX**	**FTSE**
1985	-	**0.069%**	-
1986	-	**0.148%**	-
1987	**0.236%**	**0.044%**	-
1988	**0.164%**	**0.131%**	-
1989	**0.032%**	**0.109%**	**0.091%**
1990	-0.088%	-0.220%	-0.066%
1991	-0.053%	-0.010%	**0.039%**
1992	**0.033%**	-0.108%	**0.050%**
1993	**0.102%**	**0.030%**	**0.083%**
1994	**0.073%**	**0.036%**	-0.044%
1995	-0.079%	**0.000%**	**0.070%**
1996	-0.100%	-0.028%	**0.040%**
1997	-0.224%	-0.120%	**0.059%**
1998	**0.120%**	-0.023%	**0.014%**
1999	**0.063%**	**0.068%**	**0.088%**
2000	-0.281%	-0.032%	-0.004%
2001	**0.182%**	-0.061%	-0.069%
2002	-0.052%	-0.607%	-0.132%
2003	**0.122%**	**0.097%**	**0.082%**

in UK stock market from Jan 3, 1989 to Jun 29, 2007. Because of our analysis method, analysis period for each country is different. To deduce Optimal portfolio selection rule, investor uses the past data of 1 year from initial investment and hold the portfolio for 3.5years. We assumed that there is not risk-free asset in market and investor's required rate of return is 0. And we also assume that investors invest their all wealth as a form of portfolio.

To do this analysis, we found the period which the average daily return of the each index for each country exceeded 0, and the period we found was different as <Table 1>. We conducted this method by 10 times repetitively for each country. <Table 2> represents the descriptive statistics of the sample data for each country. It shows that skewness and kurtosis of the distribution are different from normal distribution. Considering Jarque-Bera statistics and its p-value, it is clear that the sample data have non-Gaussian distribution.

As mentioned above, we assume that investor want to invest his wealth to a risky asset. Most investors in market are willing to invest with various portfolio alternatives because they have different characteristics in investment, such as risk lover, risk neutral, risk averter. Although people have different alternatives, they want expected performance of

<Table 2-1> Descrpitive Statistics of KOSPI Industrial Index in Korean Market

Index	**Mean**	**Std. Dev.**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**P-value**
Food & Beverage	0.127%	0.016	0.209	6.963	1842.121	0.000
Textiles & Wearing apparel	0.054%	0.017	-0.376	8.106	3090.340	0.000
Iron steel & metal	0.131%	0.019	0.077	7.573	2429.067	0.000
Non-metal mineral products	0.121%	0.019	0.022	5.988	1035.897	0.000
Wood & Paper	0.086%	0.019	-0.503	8.059	3085.851	0.000
Chemicals	0.124%	0.017	0.043	5.664	824.123	0.000
Medical supplies	0.101%	0.020	-0.009	8.516	3528.990	0.000
Electric&Electronics	0.134%	0.022	0.217	6.875	1763.508	0.000
Insurance	0.126%	0.024	0.181	6.375	1336.385	0.000
Banks	0.068%	0.025	0.151	5.340	646.025	0.000
Construction	0.110%	0.025	0.358	7.433	2338.985	0.000
Transport&Warehouse	0.129%	0.023	-0.140	6.900	1773.138	0.000
Machinery	0.098%	0.022	-0.123	6.085	1110.944	0.000
Securities	0.120%	0.029	0.408	6.842	1789.524	0.000
Transport equipment	0.066%	0.021	0.050	6.152	1153.530	0.000
Distribution	0.099%	0.022	0.014	5.828	927.745	0.000

portfolio to be greatest at the end of the invest period. To compare each selection rule, we consider the sample path of the final wealth obtained from the various approaches. Let W_t is the investor's wealth at time t. we assume that the investor has an initial wealth of $W_0 = 1$. And the wealth of the final day(N) is calculated as follows.

$$W_N = W_{N-1} * e^r$$

Where *r* is continuously compounded return. The results of calculated average portfolio weight is represented in <Table3> for each country. We calculated the optimal portfolio weights by 10times for 10 periods, so the values of the table represent the average value of 10 analysis periods. For each period we evaluated the rank of the performance of optimal portfolio. If we consider their ranks only at the end of the investment period, we cannot check the consistency of their perfomance during that period. Therefore, we decided that the ranks of the portfolios need to be calculated on a monthly basis in order to observe their consistency during investment period. Consequently, the number of the their ranks is

<Table 2-2> Descrpitive Statistics of TOPIX Industrial Index in Japanese Market

Index	**Mean**	**Std. Dev.**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**P-value**
RETAIL	0.096%	1.210%	-0.196	17.057	20310.595	0.000
ELEC.POWER&GAS	0.049%	1.635%	0.608	14.391	13479.226	0.000
MACHINERY	0.076%	1.077%	-1.048	19.864	29660.565	0.000
PHARMACEUTICAL	0.023%	1.189%	-0.890	21.634	35989.907	0.000
BANKS	0.084%	1.523%	0.266	14.440	13471.695	0.000
FOODS	0.046%	1.051%	-1.410	36.516	116188.532	0.000
SERVICE	0.119%	1.175%	-0.540	11.176	6985.835	0.000
INFO&COMMUNICATION	0.087%	1.833%	0.603	6.485	1396.331	0.000
MINING	0.054%	1.542%	0.415	7.138	1829.528	0.000
PULP&PAPER	0.069%	1.336%	-0.149	11.521	7466.306	0.000
PRECISIONINSTR.	0.067%	1.451%	-0.396	12.941	10213.695	0.000
OTHERPRODUCTS	0.063%	1.156%	-0.513	11.901	8244.864	0.000
CONSTRUCTION	0.071%	1.401%	-0.130	16.408	18470.667	0.000
METALPRODUCTS	0.070%	1.220%	-0.637	15.865	17166.524	0.000
IRON&STEEL	0.095%	1.737%	0.095	11.950	8230.959	0.000
FISHERIES	0.059%	1.565%	-0.026	15.774	16758.895	0.000
RUBBERPRODUCTS	0.067%	1.520%	-0.319	11.271	7067.768	0.000
OTHERFINANCIALS	0.068%	1.426%	-0.399	14.829	14437.439	0.000
WAREHOUSE	0.071%	1.428%	0.242	6.358	1182.098	0.000
OIL&COALPRODS	0.050%	1.556%	-0.067	11.970	8266.307	0.000
AIRTRANSPORT	0.068%	1.767%	0.370	9.014	3771.422	0.000
GLASS&CERAMICS	0.054%	1.246%	-0.934	23.910	45263.043	0.000
TRANSPORTEQUIP.	0.077%	1.389%	-0.429	18.381	24374.819	0.000
WHOLESALE	0.116%	1.319%	-0.472	12.786	9927.600	0.000
LANDTRANSPORT	0.080%	1.364%	-0.135	12.631	9533.718	0.000
MARINETRANSPORT	0.097%	1.807%	0.423	8.913	3664.250	0.000
SECURITIES	0.130%	2.124%	0.536	7.905	2589.500	0.000
ELECTRICMACHINE	0.082%	1.529%	-0.176	10.821	6295.779	0.000
REALESTATE	0.083%	2.111%	0.554	25.031	49975.514	0.000
NON-FERROUSMETS	0.065%	1.375%	-0.319	13.118	10556.592	0.000
TEXTILES	0.051%	1.194%	-0.788	18.620	25314.442	0.000
INSURANCE	0.068%	1.723%	0.142	11.550	7517.210	0.000
CHEMICAL	0.063%	1.131%	-1.121	23.156	42245.013	0.000

<Table 2-3> Descrpitive Statistics of FTSE Industrial Index in UK Market

Index	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Probability
Software & Computer Services	0.119%	1.220%	-1.445	24.627	50165.080	0.000
Household Goods & Textiles	0.013%	0.861%	1.357	20.711	33829.118	0.000
Real Estate	0.029%	0.828%	0.431	9.947	5164.565	0.000
Speciality & Other Finance	0.086%	0.978%	-0.346	15.458	16404.505	0.000
Health	0.049%	0.894%	0.105	10.023	5201.880	0.000
Food Producers & Processors	0.029%	0.962%	-0.494	8.993	3888.200	0.000
Investment Companies	0.074%	0.765%	-0.563	12.110	8879.299	0.000
Oil & Gas	0.057%	1.247%	0.241	5.911	917.263	0.000
Tobacco	0.063%	1.602%	1.741	28.142	67885.480	0.000
Transport	0.058%	0.921%	0.100	6.827	1547.873	0.000
Chemicals	0.029%	1.020%	-0.230	8.232	2906.885	0.000
Life Assurance	0.090%	1.479%	-0.061	8.268	2926.186	0.000
Information Technology Hardware	0.122%	2.094%	-0.098	10.192	5455.074	0.000
Telecommunication Services	0.103%	1.370%	0.322	5.631	773.294	0.000
Pharmaceuticals	0.082%	1.442%	0.121	7.680	2313.935	0.000
Construction & Building Materials	0.038%	1.016%	0.634	11.096	7076.358	0.000
Engineering & Machinery	0.053%	1.007%	0.600	11.640	8018.320	0.000
Food & Drug Retailers	0.041%	1.194%	-0.178	5.051	456.738	0.000
FTSE Leisure & Hotels	0.051%	1.064%	0.392	8.170	2880.754	0.000
Personal Care & Household Products	0.023%	1.358%	-0.997	21.340	35863.258	0.000
Electronic & Electrical Equipment	0.071%	1.376%	-1.232	32.014	89348.999	0.000
FTSE Media & Entertainment	0.081%	0.992%	0.059	6.679	1427.502	0.000

Automobiles and Parts	0.081%	1.313%	0.482	6.567	1438.256	0.000
Insurance	0.039%	1.385%	0.150	7.337	1991.931	0.000
Mining	0.060%	1.437%	0.250	6.159	1078.240	0.000
Aerospace & Defence	0.052%	1.343%	-0.269	7.839	2497.725	0.000
General Retailers	0.047%	1.020%	0.003	5.889	879.538	0.000
Support Services	0.067%	1.002%	-0.547	8.037	2799.283	0.000
Beverages	0.036%	1.327%	0.306	9.374	4321.186	0.000
Banks	0.101%	1.405%	0.388	5.844	915.383	0.000

420(=42months (for 3.5years)* 10times). The results of the ranks could be evaluated in many different ways. In this study, we use the deviation which represents differences between number of month satisfying specific conditions of the rank as below.

$$D_{Rank}^{P} = (Number_{Rank\leq 3}^{P} - Number_{Rank\geq 7}^{P})$$

Where

D_{Rank}^{P} : Deviation of Rank of Portfolio P

$Number_{Rank\leq 3}^{P}$: Number of Month when the rank of Portfolio 'P' is equal to or less than 3

$Number_{Rank\geq 7}^{P}$: Number of Month when the rank of Portfolio 'P' is equal to or greater than 7

<Table 3-1> Average Optimal portfolio weight in Korean Market

Asset	**Portfolio Weight**									mean
	Sortino	Gini	TSV	TASD	M-VaR	MAD	Minimax	Markowitz	Sharpe	
Food & Beverage	0.04	0.31	0.35	0.36	0.32	0.28	0.10	0.32	0.10	**0.24**
Textiles & Wearing apparel	0.04	0.18	0.14	0.10	0.16	0.18	0.07	0.18	0.07	**0.12**
Iron steel & metal	0.17	0.09	0.11	0.12	0.10	0.18	0.06	0.09	0.10	**0.11**
Non-metal mineral	0.10	0.05	0.08	0.06	0.07	0.05	0.06	0.06	0.13	**0.07**

products										
Wood & Paper	0.06	0.07	0.07	0.07	0.04	0.05	0.07	0.06	0.07	**0.06**
Chemicals	0.00	0.05	0.06	0.06	0.09	0.03	0.16	0.06	0.05	**0.06**
Medical supplies	0.19	0.06	0.04	0.05	0.01	0.05	0.03	0.04	0.05	**0.06**
Electric&Electronics	0.11	0.05	0.06	0.08	0.01	0.04	0.01	0.05	0.10	**0.06**
Insurance	0.09	0.02	0.02	0.03	0.02	0.01	0.20	0.03	0.08	**0.06**
Banks	0.00	0.04	0.03	0.02	0.11	0.04	0.06	0.04	0.02	**0.04**
Construction	0.03	0.02	0.02	0.01	0.04	0.01	0.11	0.02	0.04	**0.03**
Transport&Warehouse	0.02	0.03	0.02	0.04	0.02	0.04	0.03	0.02	0.04	**0.03**
Machinery	0.04	0.02	0.00	0.01	0.00	0.02	0.03	0.02	0.07	**0.02**
Securities	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	**0.02**
Transport equipment	0.00	0.01	0.00	0.00	0.01	0.02	0.00	0.01	0.00	**0.01**
Distribution	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**
Total	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

<Table 3-2> Average Optimal portfolio weight in Japanese Market

Asset	**Portfolio Weight**									mean
	Sortino	Gini	TSV	TASD	M-VaR	MAD	Minimax	Markowitz	Sharpe	
RETAIL	0.11	0.12	0.19	0.16	0.23	0.13	0.13	0.12	0.14	**0.15**
ELEC.POWER &GAS	0.00	0.15	0.13	0.13	0.10	0.15	0.08	0.14	0.01	**0.10**
MACHINERY	0.10	0.10	0.06	0.10	0.05	0.10	0.07	0.09	0.13	**0.09**
PHARMACEUTICAL	0.04	0.12	0.09	0.10	0.07	0.11	0.03	0.12	0.02	**0.08**
BANKS	0.12	0.06	0.07	0.06	0.16	0.05	0.08	0.05	0.01	**0.07**
FOODS	0.00	0.10	0.08	0.05	0.07	0.09	0.01	0.09	0.01	**0.06**
SERVICE	0.09	0.05	0.05	0.07	0.00	0.06	0.01	0.05	0.11	**0.05**
INFO&COMMUNICATION	0.02	0.01	0.04	0.01	0.13	0.01	0.16	0.02	0.01	**0.05**
MINING	0.03	0.04	0.07	0.04	0.05	0.04	0.05	0.05	0.03	**0.05**
PULP&PAPER	0.05	0.02	0.03	0.03	0.01	0.02	0.03	0.02	0.07	**0.03**
PRECISIONINSTR.	0.05	0.01	0.02	0.01	0.02	0.01	0.04	0.01	0.10	**0.03**
OTHERPRODUCTS	0.00	0.05	0.05	0.03	0.00	0.04	0.01	0.07	0.02	**0.03**

CONSTRUCTION	0.05	0.01	0.00	0.01	0.05	0.02	0.09	0.01	0.02	**0.03**
METALPRODUCTS	0.02	0.02	0.03	0.04	0.01	0.02	0.02	0.03	0.03	**0.03**
IRON&STEEL	0.10	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.10	**0.02**
FISHERIES	0.02	0.02	0.01	0.02	0.00	0.03	0.01	0.02	0.02	**0.02**
RUBBERPRODUCTS	0.00	0.02	0.01	0.02	0.02	0.02	0.01	0.03	0.01	**0.02**
OTHERFINANCIALS	0.00	0.02	0.01	0.02	0.01	0.03	0.01	0.01	0.01	**0.01**
WAREHOUSE	0.00	0.01	0.03	0.02	0.00	0.01	0.01	0.03	0.01	**0.01**
OIL&COALPRODS	0.02	0.01	0.01	0.01	0.00	0.01	0.03	0.01	0.02	**0.01**
AIRTRANSPORT	0.03	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.01	**0.01**
GLASS&CERAMICS	0.00	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.00	**0.01**
TRANSPORT EQUIP.	0.01	0.01	0.01	0.02	0.00	0.01	0.00	0.00	0.01	**0.01**
WHOLESALE	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	**0.01**
LANDTRANSPORT	0.00	0.01	0.01	0.01	0.01	0.01	0.02	0.00	0.00	**0.01**
MARINETRANSPORT	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.04	**0.01**
SECURITIES	0.03	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	**0.01**
ELECTRICMACHINE	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.02	**0.01**
REALESTATE	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**
NON-FERROUSMETS	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.00	0.00	**0.00**
TEXTILES	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	**0.00**
INSURANCE	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	**0.00**
CHEMICAL	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**
Total	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

<Table 3-3> Average Optimal portfolio weight in UK Market

Asset	**Portfolio Weight**									mean
	Sortino	Gini	TSV	TASD	M-VaR	MAD	Minimax	Markowitz	Sharpe	
Software & Computer Services	0.12	0.13	0.08	0.15	0.06	0.15	0.09	0.12	0.21	**0.12**
Household Goods & Textiles	0.15	0.13	0.11	0.10	0.14	0.13	0.03	0.12	0.11	**0.11**
Real Estate	0.04	0.14	0.18	0.15	0.13	0.15	0.02	0.15	0.04	**0.11**
Speciality & Other Finance	0.09	0.10	0.08	0.13	0.03	0.10	0.09	0.07	0.09	**0.09**
Health	0.02	0.13	0.10	0.12	0.08	0.13	0.03	0.12	0.02	**0.08**
Food Producers & Processors	0.00	0.07	0.06	0.04	0.04	0.07	0.16	0.06	0.00	**0.06**
Investment Companies	0.00	0.07	0.04	0.10	0.03	0.06	0.02	0.04	0.05	**0.04**
Oil & Gas	0.01	0.01	0.03	0.01	0.04	0.02	0.20	0.03	0.01	**0.04**
Tobacco	0.07	0.03	0.03	0.03	0.04	0.03	0.01	0.03	0.05	**0.04**
Transport	0.05	0.03	0.03	0.02	0.06	0.03	0.02	0.03	0.01	**0.03**
Chemicals	0.03	0.02	0.03	0.01	0.07	0.02	0.02	0.04	0.01	**0.03**
Life Assurance	0.06	0.00	0.01	0.00	0.02	0.00	0.12	0.00	0.03	**0.03**
Information Technology Hardware	0.09	0.01	0.01	0.02	0.00	0.01	0.00	0.01	0.08	**0.02**
Telecommunication Services	0.09	0.00	0.01	0.00	0.01	0.00	0.01	0.01	0.09	**0.02**
Pharmaceuticals	0.11	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.08	**0.02**
Construction & Building Materials	0.00	0.03	0.03	0.03	0.01	0.03	0.02	0.03	0.01	**0.02**
Engineering & Machinery	0.01	0.03	0.05	0.02	0.02	0.02	0.01	0.04	0.00	**0.02**
Food & Drug Retailers	0.02	0.01	0.02	0.01	0.04	0.01	0.02	0.02	0.01	**0.02**
FTSE Leisure & Hotels	0.00	0.01	0.02	0.01	0.06	0.01	0.01	0.02	0.02	**0.02**
Personal Care	0.00	0.00	0.02	0.00	0.03	0.01	0.04	0.01	0.00	**0.01**

& Household Products										
Electronic & Electrical Equipment	0.02	0.00	0.01	0.00	0.02	0.00	0.02	0.01	0.01	**0.01**
FTSE Media & Entertainment	0.00	0.01	0.02	0.02	0.00	0.00	0.01	0.01	0.02	**0.01**
Automobiles and Parts	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	**0.01**
Insurance	0.00	0.00	0.02	0.00	0.02	0.00	0.00	0.01	0.00	**0.01**
Mining	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.02	**0.01**
Aerospace & Defence	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.02	**0.01**
General Retailers	0.00	0.00	0.00	0.00	0.01	0.00	0.03	0.00	0.00	**0.00**
Support Services	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	**0.00**
Beverages	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	**0.00**
Banks	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	**0.00**
Total	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

The results of the evaluation for the performance of each portfolio is represented in <Table 4>. <Table 4-1> shows the results from Korean market. For 1 period(1987), M-VaR optimal portfolio's performance was best. But for 2 period(1988), Gini portfolio was best. It shows that we need to evaluate the performance of optimal portfolio repetitively for different investment period. Totally, the best optimal portfolio is TASD model and its deviation is 130. <Table 4-2> shows the results of Japanese maket. Those results are different from Korean market. The best optimal portfolio is Gini portfolio whose deviation is 226. The results from <Table 4-3> are regarding UK market. The best portfolio is TASD models, which is the same result as Korea. But the other ranks of portfolio is different. Although the results of best portfolio are different for each country, the performance of some portfolio derived from the model assuming normal distribution on asset's return such as markowitz and sharpe model is relatively low to alternative models assuming non-gaussian distribution. The value of deviation of the classic models is quite different comparing to that of the best portfolio for each country. In this study, the results showed that the performance of classic models is inferior to that of alternative models in the actual stock markets in which the return on asset was not normally distributed.

<Table 4-1> The result of monthly portfolio evaluation in Korean Market

Period	Item	TASD	TSV	MAD	Markowitz	Gini	Minimax	M-VaR	Sharpe	Sortino
Period 1 (1987)	$Rank \leq 3$	0	0	20	2	1	0	41	37	25
	$Rank \geq 7$	18	28	8	14	20	38	0	0	0
	deviation	-18	-28	12	-12	-19	-38	41	37	25
	Rank	**6**	**8**	**4**	**5**	**7**	**9**	**1**	**2**	**3**
Period 2 (1988)	$Rank \leq 3$	11	21	37	13	41	1	1	1	0
	$Rank \geq 7$	0	1	0	0	0	8	41	34	42
	deviation	11	20	37	13	41	-7	-40	-33	-42
	Rank	**5**	**3**	**2**	**4**	**1**	**6**	**8**	**7**	**9**
Period 3 (1989)	$Rank \leq 3$	12	35	0	5	1	33	9	9	22
	$Rank \geq 7$	14	4	10	19	9	0	29	29	12
	deviation	-2	31	-10	-14	-8	33	-20	-20	10
	Rank	**4**	**2**	**6**	**7**	**5**	**1**	**8**	**8**	**3**
Period 4 (1992)	$Rank \leq 3$	35	0	24	18	37	2	2	2	6
	$Rank \geq 7$	3	5	3	1	0	9	33	36	36
	deviation	32	-5	21	17	37	-7	-31	-34	-30
	Rank	**2**	**5**	**3**	**4**	**1**	**6**	**8**	**9**	**7**
Period 5 (1993)	$Rank \leq 3$	39	0	5	14	6	0	29	25	8
	$Rank \geq 7$	0	11	21	1	6	42	11	9	25
	deviation	39	-11	-16	13	0	-42	18	16	-17
	Rank	**1**	**6**	**7**	**4**	**5**	**9**	**2**	**3**	**8**
Period 6 (1994)	$Rank \leq 3$	25	2	18	5	0	20	17	18	21
	$Rank \geq 7$	14	1	0	15	20	18	18	21	19
	deviation	11	1	18	-10	-20	2	-1	-3	2
	Rank	**2**	**5**	**1**	**8**	**9**	**3**	**6**	**7**	**3**
Period 7 (1998)	$Rank \leq 3$	38	11	37	9	25	5	0	1	0
	$Rank \geq 7$	0	0	1	0	1	0	42	41	41
	deviation	38	11	36	9	24	5	-42	-40	-41
	Rank	**1**	**4**	**2**	**5**	**3**	**6**	**9**	**7**	**8**
Period 8 (1999)	$Rank \leq 3$	1	33	3	31	8	38	1	5	6
	$Rank \geq 7$	17	0	13	3	6	1	21	30	35
	deviation	-16	33	-10	28	2	37	-20	-25	-29

	Rank	**6**	**2**	**5**	**3**	**4**	**1**	**7**	**8**	**9**
Period 9 (2001)	$Rank \le 3$	3	38	2	3	1	31	33	7	8
	$Rank \ge 7$	5	0	40	1	40	6	3	0	31
	deviation	-2	38	-38	2	-39	25	30	7	-23
	Rank	**6**	**1**	**8**	**5**	**9**	**3**	**2**	**4**	**7**
Period 10 (2003)	$Rank \le 3$	37	15	31	0	16	2	20	3	2
	$Rank \ge 7$	0	5	1	6	6	32	0	37	39
	deviation	37	10	30	-6	10	-30	20	-34	-37
	Rank	**1**	**4**	**2**	**6**	**4**	**7**	**3**	**8**	**9**
Total Period	$Rank \le 3$	201	155	177	100	136	132	153	108	98
	$Rank \ge 7$	71	55	97	60	108	154	198	237	280
	deviation	130	100	80	40	28	-22	-45	-129	-182
	Rank	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

<Table 4-2> The result of monthly portfolio evaluation in Japanese Market

Period	Item	Gini	MAD	Markowitz	TASD	TSV	Sharpe	M-VaR	Minimax	Sortino
Period 1 (1985)	$Rank \le 3$	8	16	10	22	4	33	3	0	30
	$Rank \ge 7$	8	0	15	8	17	4	24	42	8
	deviation	0	16	-5	14	-13	29	-21	-42	22
	Rank	**5**	**3**	**6**	**4**	**7**	**1**	**8**	**9**	**2**
Period 2 (1986)	$Rank \le 3$	0	9	13	0	36	0	24	41	3
	$Rank \ge 7$	3	1	0	41	0	42	1	0	38
	deviation	-3	8	13	-41	36	-42	23	41	-35
	Rank	**6**	**5**	**4**	**8**	**2**	**9**	**3**	**1**	**7**
Period 3 (1987)	$Rank \le 3$	29	21	0	35	0	36	1	0	4
	$Rank \ge 7$	0	0	1	1	26	1	41	41	15
	deviation	29	21	-1	34	-26	35	-40	-41	-11
	Rank	**3**	**4**	**5**	**2**	**7**	**1**	**8**	**9**	**6**

Period 4 (1988)	$Rank \le 3$	38	28	38	4	13	4	0	0	1
	$Rank \ge 7$	0	0	0	0	3	1	41	42	39
	deviation	38	28	38	4	10	3	-41	-42	-38
	Rank	**1**	**3**	**1**	**5**	**4**	**6**	**8**	**9**	**7**
Period 5 (1989)	$Rank \le 3$	36	39	14	11	0	7	17	0	2
	$Rank \ge 7$	0	1	0	0	9	31	10	36	39
	deviation	36	38	14	11	-9	-24	7	-36	-37
	Rank	**2**	**1**	**3**	**4**	**6**	**7**	**5**	**8**	**9**
Period 6 (1993)	$Rank \le 3$	20	31	26	1	5	1	11	30	1
	$Rank \ge 7$	7	5	1	12	16	37	0	9	39
	deviation	13	26	25	-11	-11	-36	11	21	-38
	Rank	**4**	**1**	**2**	**6**	**6**	**8**	**5**	**3**	**9**
Period 7 (1994)	$Rank \le 3$	41	18	42	23	0	1	0	0	1
	$Rank \ge 7$	0	0	0	0	2	40	12	12	31
	deviation	41	18	42	23	-2	-39	-12	-12	-30
	Rank	**2**	**4**	**1**	**3**	**5**	**9**	**6**	**6**	**8**
Period 8 (1995)	$Rank \le 3$	26	0	37	0	41	9	7	5	1
	$Rank \ge 7$	0	5	0	1	0	10	32	37	41
	deviation	26	-5	37	-1	41	-1	-25	-32	-40
	Rank	**3**	**6**	**2**	**4**	**1**	**4**	**7**	**8**	**9**
Period 9 (1999)	$Rank \le 3$	37	33	0	2	0	1	32	19	2
	$Rank \ge 7$	0	0	2	33	6	41	4	1	39
	deviation	37	33	-2	-31	-6	-40	28	18	-37
	Rank	**1**	**2**	**5**	**7**	**6**	**9**	**3**	**4**	**8**
Period 10 (2003)	$Rank \le 3$	9	39	7	4	0	34	0	0	33
	$Rank \ge 7$	0	0	7	4	37	5	26	41	6
	deviation	9	39	0	0	-37	29	-26	-41	27
	Rank	**4**	**1**	**5**	**5**	**8**	**2**	**7**	**9**	**3**
Total Period	$Rank \le 3$	244	234	187	102	99	126	95	95	78
	$Rank \ge 7$	18	12	26	100	116	212	191	261	295
	deviation	226	222	161	2	-17	-86	-96	-166	-217
	Item	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

<Table 4-3> The result of monthly portfolio evaluation in UK Market

Period	Item	TASD	Minimax	MAD	TSV	Sharpe	Gini	Sortino	M-VaR	Markowitz
Period 1 (1989)	$Rank \le 3$	0	38	6	0	33	6	35	2	6
	$Rank \ge 7$	36	3	22	25	5	18	4	10	3
	deviation	-36	35	-16	-25	28	-12	31	-8	3
	Rank	**9**	**1**	**7**	**8**	**3**	**6**	**2**	**5**	**4**
Period 2 (1991)	$Rank \le 3$	15	0	39	7	1	33	2	2	27
	$Rank \ge 7$	0	28	0	13	33	1	35	9	7
	deviation	15	-28	39	-6	-32	32	-33	-7	20
	Rank	**4**	**7**	**1**	**5**	**8**	**2**	**9**	**6**	**3**
Period 3 (1992)	$Rank \le 3$	25	41	6	5	5	1	2	41	0
	$Rank \ge 7$	1	0	22	0	24	17	27	1	34
	deviation	24	41	-16	5	-19	-16	-25	40	-34
	Rank	**3**	**1**	**5**	**4**	**7**	**5**	**8**	**2**	**9**
Period 4 (1993)	$Rank \le 3$	23	34	0	32	5	13	0	12	7
	$Rank \ge 7$	11	2	12	0	26	7	35	22	11
	deviation	12	32	-12	32	-21	6	-35	-10	-4
	Rank	**3**	**1**	**7**	**1**	**8**	**4**	**9**	**6**	**5**
Period 5 (1995)	$Rank \le 3$	41	5	3	13	31	7	22	4	0
	$Rank \ge 7$	1	10	20	3	10	7	12	31	32
	deviation	40	-5	-17	10	21	0	10	-27	-32
	Rank	**1**	**6**	**7**	**3**	**2**	**5**	**3**	**8**	**9**
Period 6 (1996)	$Rank \le 3$	37	5	0	0	42	0	42	0	0
	$Rank \ge 7$	0	0	0	11	0	32	0	41	42
	deviation	37	5	0	-11	42	-32	42	-41	-42
	Rank	**3**	**4**	**5**	**6**	**1**	**7**	**1**	**8**	**9**
Period 7 (1997)	$Rank \le 3$	18	12	0	2	42	0	39	13	0
	$Rank \ge 7$	1	5	35	0	0	39	1	6	39
	deviation	17	7	-35	2	42	-39	38	7	-39
	Rank	**3**	**4**	**7**	**6**	**1**	**8**	**2**	**4**	**8**
Period 8	$Rank \le 3$	31	25	22	24	11	2	10	1	0
	$Rank \ge 7$	0	6	0	8	29	4	31	37	11

(1998)	deviation	31	19	22	16	-18	-2	-21	-36	-11
	Rank	**1**	**3**	**2**	**4**	**7**	**5**	**8**	**9**	**6**
Period 9 (1999)	$Rank \leq 3$	3	36	39	1	2	6	0	33	6
	$Rank \geq 7$	35	3	1	1	39	0	42	4	1
	deviation	-32	33	38	0	-37	6	-42	29	5
	Rank	**7**	**2**	**1**	**6**	**8**	**4**	**9**	**3**	**5**
Period 10 (2003)	$Rank \leq 3$	9	0	37	9	5	32	5	1	28
	$Rank \geq 7$	0	23	1	0	32	4	37	24	5
	deviation	9	-23	36	9	-27	28	-32	-23	23
	Rank	**4**	**6**	**1**	**4**	**8**	**2**	**9**	**6**	**3**
Total Period	$Rank \leq 3$	202	196	152	93	177	100	157	109	74
	$Rank \geq 7$	85	80	113	61	198	129	224	185	185
	deviation	117	116	39	32	-21	-29	-67	-76	-111
	Item	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

4. Conclusion

Criteria that can be used for making optimal portfolio have been invented on the basis of various theoretical assumptions. We used nine risk measures as optimal portfolio selection rule and compared their performance in stock market for 3 countries. In this study, the return on asstes are not normally distributed in common for each country. Our empirical study shows that the best optimal portfolio is different for each country. But, the performance of classic models such as sharpe ratio and markowtiz model is relatively low to alternative models. The results showed that the performance of classic models is inferior to that of alternative models in the actual stock markets in which the return on asset was not normally distributed.

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