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**R&D investment and Spillover Effect in Downstream Network Industries**

**Abstract**

This paper explores economically a vertical separated industry with vertical differentiation. Downstream firms are competing on prices and decide to invest in R&D in order to enhance their services quality involving so a spillover effect. The main results show that even if spillover increase, firms will be motivated to spend more on R&D.

**Keywords:** R&D investment, Spillovers effect, Network industries, Vertical separation.

**JEL Classification**: D43; L13; L24.

# 1 Related literature

Many frameworks have dealt with vertical cooperation between providers and retailers (G. Atallah 2000, 2005; A. Ishii 2004...) and the interest of horizontal technological cooperation between firms (C. D'Aspremont and A. Jacquemin 1988, Kamie, Muller and Zang 1992...). Whereas, the relation between vertical market structure and R&D spillover effect has received comparatively little attention. In this paper we try to study the link between downstream R&D investment and spillover effect in the case of two vertical separated industries. In particular, we try to explore the duopoly behavior in the case of price competition.

The R&D externalities or spillover imply that some benefits of each firm's R&D flow without payment to other firms. There were two ways of modelling this externality. The first one was with Ruff (1969), Spence (1984) and KMZ (1992)[1] who postulate the presence of spillover effect on R&D dollars: each firm's effective R&D investment is the sum of others firm's expenditures. The second one was held by D'Aspermont and Jacquemin (1988) where each firm's final cost reduction is the sum of its own part and fraction of all

[1] Kamien, Muller and Zang.

others firm's parts. It is always known that linkage between firms could be efficient for successful and profitable innovation both for vertical or horizontal cooperation [2]. In our study, we just consider the case of horizontal spillover between downstream firms which decide to invest in R&D to improve their quality of services without cooperation. There is many kind of spillover: market spillover, network spillover and knowledge spillover. In general, if one firm invests in R&D, this generates knowledge ledding eventually to improved products or lower production costs for this firm.

There are many frameworks that deal with spillover effect in addition to those cited before. The most that reseamble to ours are there of A. Ishii (2004) who studied the effect of cooperative R&D in two vertically related duopolies with horizontal and vertical spillovers. She considered different kind of cooperation between firms and found that depending on the nature of cooperation, there will be larger social welfare and technological improvement is accelerated with vertical R&D cartels. The previous work was an extension from that of G. Attallah (2002) who studies vertical and horizontal R&D spillovers between upstream and downstream firms with four cases of R&D cooperation. It is shown that vertical spillovers increase R&D and welfare, while horizontal spillovers may increase or decrease them.

An other research which also near to ours is those of R. Lukach, P. M. Kort and J. Plasmans (2007). They studies the R&D investment decisions of a firm facing the threat of new technology entry and considered that there is technical uncertainty. They conclude that the incumbent will always success to prevent entry by innovation bu this new entry uncertainty makes monopolist not to be the same with and without the threat of new entry. It is also shown that depending on the innovation project, there will be less welfare because of entry deterrance.

D. Leahy and J. P. Neary and J. Plasmans (2007) who consider in additon to R&D spillover effect, absorptive capacity showed that costly absorption both raises the effectiveness of own R&D and lowers the effective spillover coefficient. An other study close to ours is those of S. Buehler and A. Schmutzler (2008) where they consider Cournot successive oligopoly with

[2](see Von Hippel, 1988; Riggs and Von Hippel,1994; Lee, 1996)

endogenous vertical integration and cost-reducing downstream investment. Dealing both with vertical separation and integration, they show that the latter market structure increases own investment and decreases competitor investment. Whereas, compared to benchmark model without investment, complete vertical separation is a less likely outcome.

The plan of the paper is as follows. We first inroduce our model model where we present the timing of the game and the caracterization of the demand functions. Then, we determine equilibruim prices and R&D expenditures. In the latter subsection, we distinguish between two symmetric and asymmetric cases depending on consumers ability to pay for the product.

*The timing of the game is as follow:*

*First, the upstream firm sets the investment level $k$ and the access price $a$.*

*Second, the downstream firms strategically fix their R&D expenditures and compete in Bertrand fashion (price competition).*

# 2 The model

In this paper, we consider two vertically related industries. There is a monopoly in the upstream market which is in charge of essential facilities (infrastructure) and two firms in the downstream market which are not symmetric as they are vertically differentiated.We suppose that firm1 has better quality services than firm2 : $q_1 > q_2$[3]. Downstream firms are competing in price (à la Bertrand). As in the classical model of Mussa&Rosen (1978), consumers differ in their intensity of preference for quality. They are distributed uniformly on a segment $\left[\underline{\theta}, \overline{\theta}\right]$[4]. The parameter $\theta$ represents the consumers'taste for quality or marginal utility of quality. In the model of vertical differentiation, we reinterpret the right unit as the locus of different

[3]This assymption is crucial in our study. It could be explained by the fact that one firm has more experience and better customer loyality than the other firm having so in other terms some competitive advantage comparing to its rival..

[4]with $\underline{\theta} > 0$ with a density equal to $1((\overline{\theta} - \underline{\theta}) = 1)$.

qualities of goods. It thus represents the vertical differentiation in a model space. A consumer type $\theta$ has an utility equals to:

$$U_i = \theta q_i + v_i k - p_i$$

if he buys one unit of good of quality $q_i = e_i + \lambda e_j$; $(i, j = 1, 2 \text{ with } i \neq j)$ at price $p_i$. The parameter $e_i$ corresponds to the quality improvement of the firm $i$ : its own R&D efforts, $\lambda e_j$ is indirectly improving the spillover effect[5].

The mesure of this spillover is represented by the parameter $\lambda \in ]0, 1[$, and $v_i$ is the consumers'willingness to pay for the service[6] at the price $p_i$.

In order to discuss the important notion that there may be a benefit from having large investments in infrastructure, despite the associated costs, we introduce a parameter $k > 0$ representing the investment realized by the upstream firm that is in charge of infrastructure and which controls the input market for local access. Both the indirect utilities $v_i$ and the upstream investment $k$ are exogenous and non-negative.[7]For simplicity, we suppose that $v_1 > v_2$ and $v_2 > 0$[8].

The upstream firm sells access to downstream firms at price $a$ and decides whether to invest in infrastructure according to a level $k$. Downstream firms are symmetrics in costs as they only have to pay the access to the upstream firm $a$ (marginal cost are eaqual to zero). Both firms operate independently of he upstream market. So according from which firm customers buy services,

[5]Spillover is used to capture the idea that some of the economic benefits of R&D activities accrue to economic agents other than the party that undertakes the research.

[6]$(\theta q_i + v_i k)$ is the maximum consumers'willingness to pay for tha product, we suppose that $v$ is sufficiently high as the market is totally covered.: that hypothesis ensures that all users will get positive utility when they buy the product.

[7]Valletti&Cambini 2004 use the same parameter but they consider that the invesment level $k$ also affects the users quantities for studying the collusion possibilities between operators in the case of interconnexion networks.Whereas, in this paper indirect utilities will be independent from the price and quantites too.

[8]it means that firm1'products have an intrinsec quality which greater than that of firm2.

their utilities will be equal to :

$$\begin{cases} U_1 = \theta\left(e_1 + \lambda e_2\right) + kv_1 - p_1 \ \text{ if consumer buy from firm 1} \\ U_2 = \theta\left(e_2 + \lambda e_1\right) + kv_2 - p_2 \text{ if consumer buy from firm 2} \\ U = 0 \ \text{ if consumer buy nothing} \end{cases}$$

The idea is to see if firms will invest more or less in R&D depending of the spillover parameter $\lambda$ and the respective R&D efforts $e_i$.

Downstream firm $i = 1, 2$ produces a final service that sells to final users at price $p_i$ $(i = 1, 2)$. Downstream firms are vertically differentiated as they are investing in R&D. Each firm's R&D investment also contributes to the knowledge stockes of its rivals through spillovers. As there are two vertically related industries, spillover could also occurs between upstream and downstream firms as what we call vertical spillover.[9] But, in this paper, we will just focus on the case of horizontal spillover between downstream firms. After engaging in R&D, downstream firms incur quadratic R&D costs equal to $c(e_i) = \frac{1}{2}e_i^2$, $i = 1, 2$.[10]We suppose that firms find it best to invest in R&D so as the degree of usefulness of the knowledge created between downstream firms is higher enough.

## 2.1 The game

Both firms operate independently from the upstream market as the industry is vertically separated. The profit function is written as follows:

$$Max\ \pi_i = \left(p_i - a\right) D_i - c\left(e_i\right)$$

We assume that firms only operate in the market if they have non-negative profits as:

[9] For more focus on vertical spillover see G.Attalah (2000) and A.Ishii (2004).

[10] The quadratic R&D costs reflect the existence of diminishing returns to R&D expenditures: a valuable justification of this assumption is that technological possibilities linking R&D inputs and innovative outputs do not display any economies of scale with respect to the size of the firm in which R&D are undertaken (see Dasgupta 1986).

$$\pi_1 \geq 0, \ \pi_2 \geq 0$$

The game has two stage: access and infrastructure investment at the first stage and R&D and prices at the second stage. We solve the game by backward induction. In the first stage, the upstream firm chooses its infrastructure investment level $k$ and sets the access price $a$. In the second stage, the downstream firms strategically choose their R&D expenditures $e_i$ then compete in prices à la Bertrand taking into account infrastructure investment level $k$, access price $a$ and R&D expenditures $e_i$.

## 2.2 The demand

Generally, authors consider competition between two firms that choose simultaneously their qualities $q_i$ in $\left[\underline{q}, \overline{q}\right]$ then set their prices $p_i$. We assume that $q_1 > q_2$. Therfore two types of market configuration are possible:

- **Covered market with purchase obligation:** in this case each $\underline{\theta} > 0$ (Tirole 1988 and Anderson 1992), each consumer buys one unit of good to the firm which gives him the greatest utility , we observe two cases depending on the value of $\underline{\theta}$ and $\overline{\theta}$ :If

$$\overline{\theta} > 2\underline{\theta} \begin{cases} p_1^* = \frac{\left(2\overline{\theta} - \underline{\theta}\right)}{3}(q_1 - q_2) \\ p_2^* = \frac{\left(\overline{\theta} - 2\underline{\theta}\right)}{3}(q_1 - q_2) \\ q_1^* = \overline{q} \\ q_2^* = \underline{q} \end{cases}$$

So both firms are active: they differentiate their products up to minimize price competition.

$$\text{If } \overline{\theta} \leq 2\underline{\theta} \begin{cases} p_1^* = \underline{\theta}\,(q_1 - q_2) \\ p_2^* = 0 \end{cases} \quad \text{and the equilibruim qualities : } q_1^* = \overline{q}$$

In this case, the market is so close for both firms: only the firm with the highest quality is active in the market.

**-Partially covered market:** it has no obligation to purchase, in this case $\underline{\theta} = 0$: each consumer buys from firm 1, 2 or buy anything:

$$\text{The equilibruim prices:} \begin{cases} p_1^* = \frac{2\overline{\theta}q_1}{4q_1 - q_2}(q_1 - q_2) \\ p_2^* = \frac{\overline{\theta}q_2}{4q_1 - q_2}(q_1 - q_2) \end{cases}$$

$$\text{The markets shares:} \begin{cases} D_1^* = \frac{2\overline{\theta}q_1}{4q_1 - q_2} \\ D_2^* = \frac{\overline{\theta}q_1}{4q_1 - q_2} \end{cases}$$

$$\text{The profits at the equilibruim prices:} \begin{cases} \pi_1^* = \frac{4\overline{\theta}^2 q_1^2}{(4q_1 - q_2)^2}(q_1 - q_2) \\ \pi_2^* = \frac{\overline{\theta}^2 q_1 q_2}{(4q_1 - q_2)^2}(q_1 - q_2) \end{cases}$$

$$\text{The equilibruim qualities are equal to:} \begin{cases} q_1^* = \overline{q} \\ q_2^* = \frac{4}{7}\overline{q} \end{cases}$$

Firms are still active at the equilibruim and quality differentiation is high but not maximum. The main result that follows from such work on vertical differentiation is that firms tend to increase the quality of their products and create a differentiation degree as consumers are able to pay. This degree of differentiation varies with the market configuration (full or partial coverage). This differentiation is generally used to soften price competition between firms.

**-characterization of demand functions**

In our model, we consider homogeneous demand in the downstream or retail market. All users have a taste for the quality represented by $\theta$ wich is uniformely distributed in the population of users between $\underline{\theta}$ and $\overline{\theta}$ which are both positive (in a population which the density is 1 : so we consider the $\left(\overline{\theta} - \underline{\theta}\right) = 1$). Each user buys one unit of the final service. The customers are heterogeneous with respect to the value they attribute to the parameter $\theta$ of the "preference for quality". We consider that firms have symetrics marginal costs and equal to zero. They have only to pay access charge $a$ to the upstream firm.

$$\begin{cases} U_i = \theta\left(e_i + \lambda e_j\right) + kv_i - p_i \;\; \text{if consumer buy from firm } i \\ U = 0 \;\; \text{if consumer buy nothing} \end{cases} \tag{1}$$

The customer expected to buy at least one unit of the product from the firm that insure the best utility. The customer buys a product only and only if $p_i \leq \theta q_i + v_i k$ which is the consumers'availability to pay for quality $q_i = e_i + \lambda e_j$: the purchased price should not exceed $(\theta q_i + v_i k)$ which represents the maximum price for which consumers prefer to buy quality $q_i$ at the option of not buying anything.

We assume without loss of generality that firm 1 sells the high quality product $q_1 > q_2$. Let $\theta_1$ be the consumer indifferent between buying from firm 1 and not buying anything, he is defined by $U_1 = 0$:

$$\theta_1 = \frac{p_1 - v_1 k}{q_1}$$

Let $\theta_2$ be the consumer indifferent between buying from firm 2 and not buying anything, he is defined by $U_2 = 0$, any consumer of type $\theta < \theta_2$ does not buy anything:

$$\theta_2 = \frac{p_2 - v_2 k}{q_2}$$

We also define $\widehat{\theta}$ the marginal consumer who is indifferent between buying from firm 1 or firm 2 càd $U_1\left(\theta, q_1, q_2\right) = U_1\left(\theta, q_1, q_2\right)$:

$$\widehat{\theta} = \frac{p_1 - p_2 - v_1 k + v_2 k}{q_1 - q_2} \tag{2}$$

We note $(p_1 - p_2) = \Delta p$; $(v_1 - v_2)\,k = \Delta vk$ and $(q_1 - q_2) = \Delta q$ :

$$\widehat{\theta} = \frac{\Delta p - \Delta vk}{\Delta q}$$

It is shown that the marginal consumer depends on the R&D efforts $e_i$ and the upstream investment $k$. It is lower than the classical model of Mussa and Rosen (1978) where the marginal consumer is defined by $\frac{\Delta p}{\Delta q}$. In this model, firm 1 offering a better quality product has a greater market share.

**Lemma 1**

$$\begin{cases} \theta_1 \geq \theta_2 \Leftrightarrow \widehat{\theta} \geq \theta_i; \; i = 1, 2 \\ \theta_1 \leq \theta_2 \Leftrightarrow \widehat{\theta} \leq \theta_i; \; i = 1, 2 \end{cases} \tag{3}$$

*See Proof AppendixA.*

We obtain the following expression in function of $\theta_1, \theta_2$ and $\widehat{\theta}$ :

$$\begin{aligned} \left(\widehat{\theta} - \theta_2\right) &= \frac{q_1}{\Delta q} (\theta_1 - \theta_2) \\ \left(\widehat{\theta} - \theta_1\right) &= \frac{q_2}{\Delta q} (\theta_1 - \theta_2) \end{aligned}$$

since $q_1 > q_2 \Leftrightarrow \Delta q > 0$ :

$$\left(\widehat{\theta} - \theta_1\right) \text{ and } \left(\widehat{\theta} - \theta_2\right) \text{ follow the sign of } (\theta_1 - \theta_2)$$

.

For all $\theta \in [\underline{\theta}, \overline{\theta}]$, we have $U_1 - U_2 = \Delta q \left(\theta - \widehat{\theta}\right)$ following the sign of $\left(\theta - \widehat{\theta}\right)$. (see appendix A).

Since prices are more easily adjustables than quanities, that it is more reasonable to model the competition in price by two-stage game :

1-Choice of the upstream investment level $k$ and R&D efforts $e_i$, subsequently, choice of qualities independently and simultaneously.

2-Choice of prices: once $e_i$ and $q_i$ determined and observed by all, both firms simultaneously compete on price $p_i$ (competition à la Bertrand).

To solve this game and determine the equilibrium under perfect game, we proceed by backward induction to given levels of $e_i$ and $q_i$, we first solve the game prices (second stage) then the equilibruim expenditures.

# 3 The equilibrium

Downstream firms are vertically differentiatied. Given that $\lambda \in ]0,1[$ and $e_1 > e_2$: we first consider the case where both firms are active and can operate in the retail market togheter. Users buy from firm $i$ only and only if $U_i > U_j$ with $i,j = 1,2$ and $i \neq j$. Then, each firm maximizes its profit given that the price of its competitor. Profit functions are written as follows:

$$\pi_i = (p_i - a)\, D_i - \frac{1}{2} e_i^2 \ ; \ \ i,\, j = 1,2 \text{ and } i \neq j \tag{4}$$

If the consumer buys one unit of good quality $q_i$ at price $p_i$ : then $\theta$ represents his taste preference for quality: the more $\theta$, the higher the satisfaction the consumer derives from the quality $q_i$, then $\theta q_i$ represents the availability to pay for quality $q_i$. Consumers are uniformly distributed on the segment $\left[\underline{\theta}, \overline{\theta}\right]$ with $\underline{\theta} > 0$ according to their preference for quality. The downstream market could so be represented as a line of unit size. Let $\widetilde{\theta}$ be the consumer indifferent between the two levels of quality $q_1$ and $q_2$ so

$$\begin{aligned} U_1 &= U_2 \\ \theta\,(e_1 + \lambda e_2) + k v_1 - p_1 &= \theta\,(e_1 + \lambda e_2) + k v_1 - p_1 \end{aligned}$$

$$\widetilde{\theta} = \frac{p_1 - p_2 - v_1 k + v_2 k}{(e_1 - e_2)\,(1 - \lambda)}$$

$$\widetilde{\theta} = \frac{\Delta p - \Delta v k}{\Delta e\,(1 - \lambda)} \ \text{ with } \begin{cases} \Delta p = (p_1 - p_2) \\ \Delta v = (v_1 - v_2) \\ \Delta e = (e_1 - e_2) \end{cases} \tag{5}$$

We now that : $\Delta p > 0, \Delta v > 0, \Delta e > 0$ and $\lambda < 1$.
The following condition should be satisfied to get $\widetilde{\theta} \in \left]\underline{\theta}, \overline{\theta}\right[$ :

$$\begin{cases} D_1 = \overline{\theta} - \widetilde{\theta} > 0 \\ D_2 = \widetilde{\theta} - \underline{\theta} > 0 \end{cases} \Longrightarrow \begin{cases} \overline{\theta} > \widetilde{\theta} \\ \widetilde{\theta} > \underline{\theta} \end{cases} \tag{6}$$

This condition ensures the existence of equilibruim duopoly, it provides to both firms a postive demand functions at equilibruim: it intends that firm 1 is not powerful or efficient enough to chase away firm 2 from the market: thee vertical differentiation degree of products is higher enough as both firms are able to coexist in the market.

We suppose that the condition (6) is satisfied as users preferences are uniformely distributed: users located below $\widetilde{\theta}$ buy at firm 2 and those at the top, buy at firme1. The demand functions of firms are then written as follows:

$$\begin{aligned} D_1 &= \left(\overline{\theta}-\widetilde{\theta}\right)=\overline{\theta}-\frac{\Delta p-\Delta vk}{\Delta e\left(1-\lambda\right)} \\ D_2 &= \left(\widetilde{\theta}-\underline{\theta}\right)=\frac{\Delta p-\Delta vk}{\Delta e\left(1-\lambda\right)}-\underline{\theta} \end{aligned} \tag{7}$$

We note that the firm $i$'s demand function depends both from her own R&D efforts $e_i$ and those of its rival $e_j$ (the spillover effect $\lambda$ ):

$$\begin{aligned} \frac{\partial D_1}{\partial \lambda} &= -\frac{\Delta p-\Delta vk}{\Delta e\left(1-\lambda\right)^2}<0 \\ \frac{\partial D_2}{\partial \lambda} &= \frac{\Delta p-\Delta vk}{\Delta e\left(1-\lambda\right)^2}>0 \end{aligned}$$

We refind the classical result that the spillover parameter $\lambda$ affects the firms'decisions to invest in R&D: when both firms choose egal R&D investment efforts $e_1 = e_2$, there will be any spillover effect on firms demand functions. Whereas and as we have supposed previously $e_1 > e_2 \Rightarrow \Delta e > 0 \Rightarrow$ an increase of the spillover parametre $\lambda$ implies a decrease of the demand function of firm1 and an increase of that of firm2 through a transfer of a demand from firm1 to firm2.

Let see now the impact of R&D investment efforts on demand functions:

$$\begin{aligned} \frac{\partial D_1}{\partial e_1} &= \frac{\partial D_2}{\partial e_2}=\frac{\Delta p-\Delta vk}{\Delta e^2\left(1-\lambda\right)}>0 \\ \frac{\partial D_1}{\partial e_2} &= \frac{\partial D_2}{\partial e_1}=-\frac{\Delta p-\Delta vk}{\Delta e^2\left(1-\lambda\right)}<0 \end{aligned}$$

The demand function of each firm increases with its own R&D investment efforts and decreases with those of its rivals. The impact of price strategies on demand functions is illustrated as follows:

$$\begin{aligned}\frac{\partial D_1}{\partial p_1} &= \frac{\partial D_2}{\partial p_2} = -\frac{1}{\Delta e\left(1-\lambda\right)} < 0 \\ \frac{\partial D_1}{\partial p_2} &= \frac{\partial D_2}{\partial p_1} = \frac{1}{\Delta e\left(1-\lambda\right)} > 0\end{aligned}$$

The demand of each firm decreases with its own price and increases with that of its rival (price elasticity of demand).

## 3.1 The equilibruim price:

Each firm maximizes its profits given the rival's price. The profit functions are written as follows:

$$\pi_i = \left(p_i - a\right) D_i - c\left(e_i\right)$$

At this stage of the game, we can determine the equilibruim price on the downstream market:

$$\begin{aligned}\pi_1 &= \left(p_1 - a\right)\left(\overline{\theta} - \frac{\Delta p - \Delta vk}{\Delta e\left(1-\lambda\right)}\right) - \frac{1}{2}e_1^2 \\ \pi_2 &= \left(p_2 - a\right)\left(\frac{\Delta p - \Delta vk}{\Delta e\left(1-\lambda\right)} - \underline{\theta}\right) - \frac{1}{2}e_2^2\end{aligned} \tag{8}$$

The first order conditions imply:

$$\frac{\partial \pi_1}{\partial p_1} = 0; \qquad \frac{\partial \pi_2}{\partial p_2} = 0$$

$$\begin{aligned}p_1\left(p_2\right) &= \frac{p_2 + \Delta vk + \overline{\theta}\Delta e\left(1-\lambda\right) + a}{2} \\ p_2\left(p_1\right) &= \frac{p_1 - \Delta vk - \underline{\theta}\Delta e\left(1-\lambda\right) + a}{2}\end{aligned}$$

$p_1(p_2)$ and $p_2(p_1)$ are the best response functions of both firms: they are both increasing, we have so complementary strategies.

For given $v_i$, upstream investment $k$ and using demand functions as defined in (7), the Nash-equilibrium prices after maximizing profits are equal to:

$$\begin{aligned} p_1^C &= \frac{1}{3}\left(\Delta e\,(1-\lambda)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta v k\right)+a \\ p_2^C &= \frac{1}{3}\left(\Delta e\,(1-\lambda)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta v k\right)+a>a \end{aligned} \tag{9}$$

to ensure the existence of the duopoly model, we need to impose restrictions on the parameters to satisfy provided condition (6). After substituting equilibruim prices $p_1^C$ and $p_2^C$ in condition(6) and simplifying, we obtain the following constraint:

$$\left(\overline{\theta}-2\underline{\theta}\right)>\frac{\Delta v k}{\Delta e\,(1-\lambda)} \tag{10}$$

So we deduce the following conditions on prices:

$$H_1 \Rightarrow p_2^C>a \Rightarrow \left(\overline{\theta}-2\underline{\theta}\right)>\frac{\Delta v k}{\Delta e\,(1-\lambda)} \tag{11}$$

$$H_2 \Rightarrow p_1^C>p_2^C \Rightarrow \left(\overline{\theta}+\underline{\theta}\right)>0 \tag{12}$$

*See Appendix B.*

The equilibruim quantities are equal to:

$$\begin{aligned} D_1^C &= \frac{1}{3}\frac{\left(\Delta e\,(1-\lambda)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta v k\right)}{\Delta e\,(1-\lambda)} \\ D_2^C &= \frac{1}{3}\frac{\left(\Delta e\,(1-\lambda)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta v k\right)}{\Delta e\,(1-\lambda)} \end{aligned} \tag{13}$$

$D_2^C>0$ given $H_1$. (We can verify that the market is totally covered as $D_1^C+D_2^C=1$: all customers buy the product).

It is clear that $p_2^C < p_1^C$ as users prefer to buy the best quality's product for a higher price $\left(p_2^C > 0,\, p_1^C > 0\right)$.

*See Appendix B.*

Given the equilibruim quantities and prices, the equilibrium profits are equal to:

$$\begin{aligned} \pi_1^C &= \frac{1}{9}\frac{\left(\Delta e\,(1-\lambda)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta vk\right)^2}{\Delta e\,(1-\lambda)}-\frac{1}{2}e_1^2 \\ \pi_2^C &= \frac{1}{9}\frac{\left(\Delta e\,(1-\lambda)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta vk\right)^2}{\Delta e\,(1-\lambda)}-\frac{1}{2}e_2^2 \end{aligned} \tag{14}$$

We verify that:

$$\begin{aligned} p_1^C &> p_2^C \\ D_1^C\left(p_1^C, p_2^C\right) &> D_2^C\left(p_1^C, p_2^C\right) \\ \pi_1^C &> \pi_2^C \end{aligned}$$

We note that the firm which produces the best quality is promoted on the market: profits are increasing in $\Delta e$ :

for $\Delta e = 0 \Leftrightarrow e_1 = e_2 \Rightarrow \pi_1^C = \pi_2^C$.Equilibruim prices are written as folow:

$$\begin{cases} p_1^C = a + \frac{\Delta vk}{3} \\ p_2^C = a - \frac{\Delta vk}{3} \end{cases}$$

## 3.2 The equilibruim R&D expenditures:

In this section, we try to determine the firms R&D investment expenditures and the spillover effect. At this step of the game, both firms choose their

R&D invesment efforts in order to enhance their services quality.

$$\pi_1^C = \frac{1}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta vk\right)^2}{\Delta e\left(1-\lambda\right)} - \frac{1}{2}e_1^2$$

$$\pi_2^C = \frac{1}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta vk\right)^2}{\Delta e\left(1-\lambda\right)} - \frac{1}{2}e_2^2$$

As $\left(\overline{\theta}-2\underline{\theta}\right) > \frac{\Delta vk}{\Delta e(1-\lambda)}$ and $\left(\overline{\theta}+\underline{\theta}\right) > 0$ so $\pi_1^C > 0$ and $\pi_2^C > 0$.

From equilibruim profits $\pi_1^C$ and $\pi_2^C$, we can determine the equilibruim R&D investment efforts $e_1^*$ and $e_2^*$. Indeed, if $e^*$ is the previous solution of the maximization of profits equilibruim, it must satisfy the CPO:

$$\frac{\partial \pi_1^C}{\partial e_1} = 0; \qquad \frac{\partial \pi_2^C}{\partial e_2} = 0 \tag{15}$$

By resolving the condition (14) to $e_1$ and $e_2$, we obtain $(e_1^*, e_2^*)$ wich is the Nash equilibruim of R&D investment efforts[11]:

$$\frac{\partial \pi_1^C}{\partial e_1} = \frac{2}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta vk\right)\left(2\overline{\theta}-\underline{\theta}\right)}{\left(e_1-e_2\right)} - \frac{1}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta vk\right)^2}{\Delta e^2\left(1-\lambda\right)} - e_1 \tag{16}$$

and

$$\frac{\partial \pi_2^C}{\partial e_2} = -\frac{2}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta vk\right)\left(\overline{\theta}-2\underline{\theta}\right)}{\left(e_1-e_2\right)} + \frac{1}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta vk\right)^2}{\Delta e^2\left(1-\lambda\right)} - e_2 \tag{17}$$

This imply :

[11] The second order and stability conditions of equilibruim are also checked : $\frac{\partial^2 \pi_i}{\partial e_i} < 0$.

$$e_1 = \frac{2}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta vk\right)\left(2\overline{\theta}-\underline{\theta}\right)}{\left(e_1-e_2\right)} - \frac{1}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta vk\right)^2}{\Delta e^2\left(1-\lambda\right)}$$

and

$$e_2 = -\frac{2}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta vk\right)\left(\overline{\theta}-2\underline{\theta}\right)}{\left(e_1-e_2\right)} + \frac{1}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta vk\right)^2}{\Delta e^2\left(1-\lambda\right)}$$

Let note:

$$f_1\left(e_1^*, e_2^*, \lambda\right) = \ e_1^* \ ; \ f_2\left(e_1^*, e_2^*, \lambda\right) = e_2^* \text{ with } \ i = 1, 2$$

So

$$\begin{aligned} f_1\left(e_1, e_2, \lambda\right) &= \frac{2}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta vk\right)\left(2\overline{\theta}-\underline{\theta}\right)}{\left(e_1-e_2\right)} \\ &\quad -\frac{1}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta vk\right)^2}{\Delta e^2\left(1-\lambda\right)} \end{aligned} \tag{18}$$

and

$$\begin{aligned} f_2\left(e_1, e_2, \lambda\right) &= -\frac{2}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta vk\right)\left(\overline{\theta}-2\underline{\theta}\right)}{\Delta e} \\ &\quad +\frac{1}{9}\frac{\left(\Delta e\left(1-\lambda\right)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta vk\right)^2}{\Delta e^2\left(1-\lambda\right)} \end{aligned} \tag{19}$$

We are going to observe now how the spillover parametre $\lambda$ affects the R&D investment efforts. For that, we determine $\frac{\partial e_1^*}{\partial \lambda}$ and $\frac{\partial e_2^*}{\partial \lambda}$ that could be estimated by the the following linear application:

$$\begin{cases} \frac{\partial f_1}{\partial e_1}\frac{de_1^*}{d\lambda} + \frac{\partial f_1}{\partial e_2}\frac{de_2^*}{d\lambda} + \frac{\partial f_1}{\partial \lambda} = \frac{de_1^*}{d\lambda} \\ \\ \frac{\partial f_2}{\partial e_1}\frac{de_1^*}{d\lambda} + \frac{\partial f_2}{\partial e_2}\frac{de_2^*}{d\lambda} + \frac{\partial f_2}{\partial \lambda} = \frac{de_2^*}{d\lambda} \end{cases}$$

We rewrite it under matrix shape:

$$\begin{pmatrix} \left(\frac{\partial f_1}{\partial e_1} - 1\right) & \frac{\partial f_1}{\partial e_2} \\ \frac{\partial f_2}{\partial e_1} & \left(\frac{\partial f_2}{\partial e_2} - 1\right) \end{pmatrix} \begin{pmatrix} \frac{de_1^*}{d\lambda} \\ \\ \frac{de_1^*}{d\lambda} \end{pmatrix} = \begin{pmatrix} -\frac{\partial f_1}{\partial \lambda} \\ \\ -\frac{\partial f_2}{\partial \lambda} \end{pmatrix}$$

We need so to calculate the corresponding derivatives of $f_1(e_1, e_2, \lambda)$ and $f_2(e_1, e_2, \lambda)$ :

$$\frac{\partial f_1(e_1, e_2, \lambda)}{\partial e_1} = \frac{\partial f_2(e_1, e_2, \lambda)}{\partial e_2} = \frac{2}{9}\frac{\Delta v k^2}{\Delta e^3 (1-\lambda)} > 0 \tag{20}$$

$$\frac{\partial f_1(e_1, e_2, \lambda)}{\partial e_2} = \frac{\partial f_2(e_1, e_2, \lambda)}{\partial e_1} = -\frac{2}{9}\frac{\Delta v k^2}{\Delta e^3 (1-\lambda)} < 0 \tag{21}$$

$$\frac{\partial f_1(e_1, e_2, \lambda)}{\partial \lambda} = -\frac{1}{9}\left(\frac{\Delta v k^2}{\Delta e^2 (1-\lambda)^2} + \left(2\overline{\theta} - \underline{\theta}\right)^2\right) < 0 \tag{22}$$

$$\frac{\partial f_2(e_1, e_2, \lambda)}{\partial \lambda} = \frac{1}{9}\left(\frac{\Delta v k^2}{\Delta e^2 (1-\lambda)^2} + \left(\overline{\theta} - 2\underline{\theta}\right)^2\right) > 0 \tag{23}$$

*See Appendix C.*

We use la régle de $CRAMER$ to solve this system. We deduce from the previous $\frac{de_1^*}{d\lambda}$ and $\frac{de_2^*}{d\lambda}$:

$$\frac{de_1^*}{d\lambda} = \frac{-\left(\frac{\partial f_1}{\partial \lambda}\right)\left(\frac{\partial f_2}{\partial e_2} - 1\right) + \left(\frac{\partial f_1}{\partial e_2}\frac{\partial f_2}{\partial \lambda}\right)}{\left(\frac{\partial f_1}{\partial e_1} - 1\right)\left(\frac{\partial f_2}{\partial e_2} - 1\right) - \left(\frac{\partial f_1}{\partial e_2}\frac{\partial f_2}{\partial e_1}\right)} \tag{24}$$

$$\frac{de_2^*}{d\lambda} = \frac{-\left(\frac{\partial f_2}{\partial \lambda}\right)\left(\frac{\partial f_1}{\partial e_1} - 1\right) + \left(\frac{\partial f_1}{\partial \lambda}\frac{\partial f_2}{\partial e_1}\right)}{\left(\frac{\partial f_1}{\partial e_1} - 1\right)\left(\frac{\partial f_2}{\partial e_2} - 1\right) - \left(\frac{\partial f_1}{\partial e_2}\frac{\partial f_2}{\partial e_1}\right)}$$

After substituting $\frac{\partial f_i}{\partial e_i}$, $i = 1, 2$, we obtain:

$$\begin{aligned}
\frac{de_1^*}{d\lambda} &= \frac{\Delta e^3 (1-\lambda)^2 \left(2\overline{\theta} - \underline{\theta}\right)^2 + 6\Delta v k^2 (1-\lambda) \left(\overline{\theta}^2 - \underline{\theta}^2\right) - \Delta v k^2 \Delta e}{(1-\lambda)\left(9\Delta e^3 (1-\lambda) - 4\Delta v k^2\right)} \\
\frac{de_2^*}{d\lambda} &= \frac{\Delta e^3 (1-\lambda)^2 \left(\overline{\theta} - 2\underline{\theta}\right)^2 + 6\Delta v k^2 (1-\lambda) \left(\overline{\theta}^2 - \underline{\theta}^2\right) + \Delta v k^2 \Delta e}{(1-\lambda)\left(9\Delta e^3 (1-\lambda) - 4\Delta v k^2\right)}
\end{aligned} \tag{25}$$

$$\frac{de_i^*}{d\lambda} > 0 \text{ if and only if } \Delta v k \in \left]0, \widetilde{\Delta v k}\right[$$

where $\widetilde{\Delta v k} = \frac{3}{2}\sqrt{\Delta e^3 (1-\lambda)}$.

*See proof AppendixE.*

Given the respectives expression of $\frac{de_1^*}{d\lambda}$ and $\frac{de_2^*}{d\lambda}$, we can distinguish two symmetric and asymmetric cases depending on the value of the consumers'willingness to pay for the product $v_i$.

### 3.2.1 Symmetric case $v_1 = v_2$ :

In this case, we found that prices and demands are always depending on $\lambda$:

$$\begin{aligned}
p_1^C &= \frac{1}{3}\left(\Delta e (1-\lambda) \left(2\overline{\theta} - \underline{\theta}\right)\right) + a \\
p_2^C &= \frac{1}{3}\left(\Delta e (1-\lambda) \left(\overline{\theta} - 2\underline{\theta}\right)\right) + a \\
D_1^C &= \frac{\left(2\overline{\theta} - \underline{\theta}\right)}{3}; \quad D_2^C = \frac{\left(\overline{\theta} - 2\underline{\theta}\right)}{3}
\end{aligned}$$

In this case, we remark that the spillover parametre $\lambda$ has negatif effect on firms price wheras it does'nt affect demand as:

$$\begin{aligned}\frac{\partial p_1^C}{\partial \lambda} &= -\frac{\left(2\overline{\theta}-\underline{\theta}\right)\Delta e}{3}<0\\ \frac{\partial p_2^C}{\partial \lambda} &= -\frac{\left(\overline{\theta}-2\underline{\theta}\right)\Delta e}{3}<0\\ \frac{\partial D_i^C}{\partial \lambda} &= 0\end{aligned}$$

$\lambda$ has positif effect on $e_1^*$ and $e_2^*$.

$$\left.\frac{de_1^*}{d\lambda}\right|_{\Delta v=0}=\frac{\left(2\overline{\theta}-\underline{\theta}\right)^2}{9}>0;\quad \left.\frac{de_2^*}{d\lambda}\right|_{\Delta v=0}=\frac{\left(\overline{\theta}-2\underline{\theta}\right)^2}{9}>0$$

**Proposition 2** *when* $v_1 = v_2$ *:*

$(i)$firms equilibruim R&D efforts $e_1^*$ and $e_2^*$ increase with $\lambda$.

$(ii)$firm 1 provides more efforts than firm 2 to improve the quality of services as the spillover parametre is high.

From this, we deduce that the firm $i$ , $(i = 1, 2)$ R&D investment efforts enhance her services quality but not necessarily its competitive advantage since efforts also benefit its rival (firm $j$, $j = 1, 2$ and $i \neq j$).

Firm $j$ gets so an indirect increase of its own quality services which allows it to reduce his R&D efforts$\Rightarrow$ that is spillover effect. Normally, the firm $i$ reaction toward this competition decline is to decrease its R&D efforts too. Whereas, we find that even if $\lambda$ decreases $\Rightarrow$ $e_1^*$ and $e_2^*$ increase which leads to conclude that the R&D investment efforts $e_i^*$ are strategic substitutes. So it is shown that in this case the strategic effect dominates the spillover one and even if $\lambda$ increases, the R&D investment efforts $e_i$ increase too.

### 3.2.2 Asymmetric case $v_1 \neq v_2$ :

We have shown in Proposition(1) that an increase in the spillover prametre $\lambda$ induces an increase in the equilibruim R&D efforts. However, given the firms asymmetry, the variation in the quality degrees is not so similar: the increase of $\lambda$ more favors then one of two firms in the duopoly.

**Proposition 3** *When $v_1 \neq v_2$, the firm equilibruim R&D investment efforts $e_i^*$ are:*

$(i)$ increasing with $\lambda$ only when $\Delta v$ is relatively low as $\Delta v \in \left]0, \widetilde{\Delta v}\right[$.

$(ii)$ decreasing with $\lambda$ when $\Delta v$ is relatively high as $\Delta v > \widetilde{\Delta v}$.

*Proof: see appendix E.*

This proposition shows that for a given asymmetric level $\Delta v$ relatively low: $\Delta v \in \left]0, \widetilde{\Delta v}\right[$ : firms R&D efforts $e_i$ are increasing with $\lambda$. The intuition from this result is as follows: when there is no spillover effect $\lambda = 0$ and $\Delta v \in \left]0, \widetilde{\Delta v}\right[$, the R&D efforts are perfectly adaptable: firm 2 provides more efforts as $q_2 < q_1$, she earns more in improving its services quality. In this case, it is less profitable for firm1 to imitate firm2 as its services'intrinsic quality is already high. Thus, firm1 increases its R&D investments but slightly lower than firm2. This result is true even if the spillover parametre $\lambda$ is low ($\lambda \to 0$).

So for $\Delta v \in \left]0, \widetilde{\Delta v}\right[$ :

$$\left.\frac{de_1^*}{d\lambda}\right|_{\lambda=0} = \frac{\Delta e^3 \left(2\overline{\theta} - \underline{\theta}\right)^2 + 6\Delta v k^2 \left(\overline{\theta}^2 - \underline{\theta}^2\right) - \Delta v k^2 \Delta e}{9\Delta e^3 - 4\Delta v k^2} > 0$$

$$\left.\frac{de_2^*}{d\lambda}\right|_{\lambda=0} = \frac{\Delta e^3 \left(\overline{\theta} - 2\underline{\theta}\right)^2 + 6\Delta v k^2 \left(\overline{\theta}^2 - \underline{\theta}^2\right) + \Delta v k^2 \Delta e}{9\Delta e^3 - 4\Delta v k^2} > 0$$

However, when $\lambda$ is high ($\lambda \to 1$) and $\Delta v \in \left]0, \widetilde{\Delta v}\right[$, the R&D efforts are hardly adaptable. In order to neutralize this effect, firm 1 reduces slightly the frequency or the speed of its R&D investment $e_1$ and prefers to free rides its rival (firm2). Thus, given its higher initial market share, firm1 decreases more than proportional its speed of investment in R&D. Anticipating this, firm2 increases its R&D efforts $e_2$ $\Rightarrow$there exists a non monotonic relationship between $e_i$ and $\lambda$.

In the second case, when $\Delta v > \widetilde{\Delta v}$, firms R&D efforts $e_i$ still always decreasing whatever the value of $\lambda$ ($\lambda \rightarrow 0$ or 1), precisely:

$$\frac{de_i^*}{d\lambda} < 0 \text{ if and only if } \Delta v > \widetilde{\Delta v} \text{ and } \lambda \in ]0, 1[$$

Spillover effect minimizes the firm'profits as it will be a part transferred to the rival. In the case with none spillover $\lambda = 0$, firms will enjoy the all benefit from investing in R&D.

For $\Delta v > \widetilde{\Delta v}$, both downstream firms will be less motivated or intended to invest in R&D as the spillover effect increases. The main reason that could explain such a behavior is that firm1 spends more efforts in R&D (as $e_1 > e_2$) having so a competitve advantge and refuses to share her own benefits from such investment with other firms. It could be asked here about possible case of forclusion.

As firm2 makes less efforts in R&D, it will be more profitable for that firm to keep the same R&D investment level and to take advantage from those made by firm1 through the spillover effect $\lambda$.

When $\lambda$ is near to 0, we note that downstream firms still maintaining

their R&D expenditures and as shown before, firm1 will be less intended to invest in R&D than firm2.We conclude that even if there is no spillover effect, firm 1 invest less in R&D which points the fact that firm'R&D investment incentives are not only relied to spillover effect but to degree of asymmetry between firms.

*See Appendix E.*

We can reverse this result by calculating the following comparative statics:

**Corollary 4** *the disparity between* $\frac{de_1^*}{d\lambda}$ *and* $\frac{de_2^*}{d\lambda}$ *is decreasing in* $\lambda$*:*

$$\frac{de_1^*}{d\lambda}-\frac{de_2^*}{d\lambda}=\frac{3\Delta e^3\left(1-\lambda\right)^2\left(\overline{\theta}-2\underline{\theta}\right)^2-2\Delta vk^2\Delta e}{\left(1-\lambda\right)\left(9\Delta e^3\left(1-\lambda\right)-4\Delta vk^2\right)}$$
$$\frac{\partial\Delta\left(de^*\right)}{\partial\lambda}>0$$

*See Appendix E.*

We deduce that the spillover effect reduces the disparity between R&D investment efforts of firms 1 and 2.

*See Appendix* D.

Let see now the effect of spillover parametre on prices and demand:

$$\begin{aligned}p_1^C &= \frac{1}{3}\left(\Delta e\left(1-\lambda\right)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta vk\right)+a \\ p_2^C &= \frac{1}{3}\left(\Delta e\left(1-\lambda\right)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta vk\right)+a>a\end{aligned} \tag{26}$$

$$\begin{aligned}D_1^C &= \frac{1}{3}\frac{\left(\Delta e\left(1-\lambda\right)\left(2\overline{\theta}-\underline{\theta}\right)+\Delta vk\right)}{\Delta e\left(1-\lambda\right)} \\ D_2^C &= \frac{1}{3}\frac{\left(\Delta e\left(1-\lambda\right)\left(\overline{\theta}-2\underline{\theta}\right)-\Delta vk\right)}{\Delta e\left(1-\lambda\right)}\end{aligned} \tag{27}$$

**Proposition 5** *the equilibruim prices and demand decrease with the spillover parametre* $\lambda$ *for both firms whatever* $\Delta v$*:*

$$\begin{aligned}\frac{dp_1^C}{d\lambda} &= -\frac{\Delta e\left(2\overline{\theta}-\underline{\theta}\right)}{3}<0 \\ \frac{dp_2^C}{d\lambda} &= -\frac{\Delta e\left(\overline{\theta}-2\underline{\theta}\right)}{3}<0\end{aligned}$$

$$\frac{dD_1^C}{d\lambda}=\frac{dD_2^C}{d\lambda}=-\frac{1}{3}\frac{\Delta vk}{\Delta e\left(1-\lambda\right)^2}<0$$

In this case, firms prices $p_i$ and $D_i$ demands decrease as vertical differentiation is maintained and firms are not symmetric ($v_1 \neq v_2$ and $e_1 \neq e_2$) whatever $\Delta v$ and $\lambda$.

**Corollary 6** *the price and demand difference or disparity decreases with spillover effect* $\lambda$ :

$$\begin{aligned}\Delta p^C &= \frac{\Delta e\,(1-\lambda)\left(\overline{\theta}+\underline{\theta}\right)+2\Delta vk}{3} > 0\\ \frac{\partial \Delta p^C}{\partial \lambda} &= -\frac{\Delta e\left(\overline{\theta}+\underline{\theta}\right)}{3} < 0\end{aligned}$$

$$\begin{aligned}\Delta D_1^C &= \frac{\Delta e\,(1-\lambda)\left(\overline{\theta}+\underline{\theta}\right)+2\Delta vk}{3} > 0\\ \frac{\partial \Delta D^C}{\partial \lambda} &= -\frac{\Delta e\left(\overline{\theta}+\underline{\theta}\right)}{3} < 0\end{aligned}$$

We note that the spillover parametre $\lambda$ reduces strictly the equilibruim price and demand disparity.

Let now determine the impact of the spillover parameter on $\frac{de_1^*}{d\lambda}$ and $\frac{de_2^*}{d\lambda}$ for $\lambda$ close to 0 :

$$\left.\frac{de_1^*}{d\lambda}\right|_{\lambda=0} - \left.\frac{de_2^*}{d\lambda}\right|_{\lambda=0} = \frac{3\Delta e^3\,(1-\lambda)^2\left(\overline{\theta}-2\underline{\theta}\right)^2 - 2\Delta vk^2\Delta e}{(1-\lambda)\left(9\Delta e^3\,(1-\lambda)-4\Delta vk^2\right)}$$

$$\left.\frac{\partial \Delta\left(\frac{de^*}{d\lambda}\right)}{\partial \lambda}\right|_{\lambda=0} > 0$$

# 4 Conclusion

We have tried through this framework to explore duopoly firms behavior, investing in R&D, in a case of verical separation with price competition. We have shown that as all investment study, spillover effect still influence firms prices and demands. We found that even if we have supposed that firms R&D investment expenditures increase with the spillover parametre. Whereas as their prices and demands are decreasing for an increase of spillover. This results were obtained separately in symmetric and asymmetric cases.

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