Discussion Paper

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# Walras' Law and the Problem of Money Price Determinacy

**Abstract:** In this paper I analyze the historical treatment of the problem of money price determinacy starting with Walras (1874) and Patinkin (1948) up to the latest research by Weil (1991) and Benassy (2007). I show based on an extremely simple model that Walras' Law does hold in a pure barter economy but not in a monetary economy. As a consequence, in a monetary economy with N goods there are N independent market equilibrium conditions available to determine the N money prices of all goods. For many classes of models this equality of equations and unknown variables is sufficient to determine the money prices of goods unambiguously – even in the case of Ricardian economies, where money holdings are no net wealth.

**Keywords:** Walras Law, Price Determinacy, Neoclassical Dichotomy, Real-Balance Effect, Pigou Effect, Patinkin Controversy, Inside Money, Outside Money, Monetary Theory

**JEL-Classification:** E5, B12, B13, B22

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## Content

## 1. Introduction

Since Walras (1874) has shown that the keeping of budget constraints implies an equilibrium on the $n^{th}$ market, if n-1 markets are in equilibrium, the difficulty of unambiguously determining the money prices of goods has emerged.[2] The first trial solution to this problem was the so called neoclassical-dichotomy approach proposed by authors like Walras himself (1874, p. 325), Fisher (1911, pp. 163-164) and Pigou (1917, pp. 163-164). According to this approach, the economy can be divided in two sectors: A *real sector*, described by the excess demand equations for goods and a *monetary sector*, described by the excess demand equation for money. The excess demand equations of the real sector can then be used to determine the equilibrium values of the *relative prices* (in terms of an arbitrarily chosen numéraire) of all goods, while the excess demand equation for money can be used to determine the *money price* of the numéraire. Multiplying the money price of the numéraire with the relative prices of the other goods, yields then the money prices of all goods. This neoclassical-dichotomy approach is de facto still in widespread use in monetary policy models. In monetary DSGE models, where a monetary aggregate is used as policy instrument, the standard procedure is to eliminate one market by Walras' Law and to add a money market equation, where money demand, based on a cash-in-advance or money-in-the-utility-function approach, is equalized with money supply. The latter is typically modeled as a direct money transfer into the household budget constraint.[3]

[2] Throughout this paper, I refer to "money price determinacy" as the determinacy of prices in presence of exogenous money supply. If the supply of money is endogenized dependent on other macroeconomic variables ("monetary policy rules"), it is of course possible that instable dynamic feedback-mechanisms may evolve that cause money price indeterminacy even though market mechanisms were able to unambiguously determine prices in the presence of an exogenous money supply. For a discussion of various forms of price indeterminacy generated by endogenous money supply, see McCallum (1986). To avoid misunderstandings: A nominal interest rate peg by the central bank in form of an exogenous AR(1) process like $i_t = \rho\, i_{t-1} + \varepsilon_t$, that is known to cause multiple equilibria in certain DSGE models (Bullard and Mitra (2002)) and hence a type of indeterminacy, may be called "exogenous policy rule" to distinguish it from other "more active" policy rules, but it does certainly not imply an exogenous supply of money. It is easy to see that it implies in fact a pro-cyclical money supply, since it implies a monetary accommodation of inflation expectations.

[3] Prototype models that follow this approach are the monetary RBC model of Hansen and Cooley (1989) and the staggered price-setting model of Calvo (1983).

However, as Patinkin (1965) has shown, this procedure is *self-contradictory*:[4] Starting with a general market equilibrium, a doubling of the money prices of all goods leaves the relative prices of these goods unchanged (since the money price of the numéraire doubles too). This however means that the arguments of the excess demand equations in the real sector do not change. Consequently, the markets of the real sector of the economy stay in equilibrium. Nevertheless, the doubling of money prices will half real money supply. Following the neoclassical-dichotomy approach, the resulting excess demand for money would then "somehow" cause the money prices of goods to fall back to their equilibrium values and thus keep the money prices at a well determined level. However, by Walras' Law, the latter cannot happen, since, if all other markets stay in equilibrium, the money market too must stay in equilibrium. Therefore an excess demand for money cannot emerge. Therefore, by Walras' Law, the level of the money prices of goods stays indeterminate: No matter by what factor the money prices of all goods are multiplied, the relative prices of goods stay constant and therefore the equilibrium on the goods markets stays unchanged and hence, by Walras' Law, no disequilibrium on the money market, which causes money prices of goods to return to their old values, can emerge.

Patinkin's way out of this dilemma is the introduction of a wealth effect, which he first calls "Pigou Effect" (Patinkin (1948)) and then "real-balance-effect" (Patinkin (1965)). He introduces this wealth effect by adding the real value of money balances as an additional, positively valued argument in the excess demand equations. In other words, Patinkin takes care for the changes of the real value of the money holdings caused by a change of money prices. As a consequence, a doubling of the money prices of all goods does, as before, not affect the relative prices of goods, but it does now reduce the value of money holdings by half and, this way, causes an excess supply of goods, which, by Walras' Law corresponds to an excess demand for money. These disequilibria, in turn, cause the price level to fall back to its

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[4] In Patinkin's words (1965, p. 176): "For let the assumptions of the dichotomy obtain. Assume now that an initial position of equilibrium is disturbed in such a way as to cause an equiproportionate change in all money prices. Since this does not change relative prices, the "homogeneity postulate" implies that none of the demand functions in the real sector are thereby affected. Hence, since the commodity markets of this sector were initially in equilibrium, they must continue to be so. *By Walras' Law, so must the money market.* Thus the equiproportionate departure of money prices from any given equilibrium level creates no market forces – that is, creates no amounts of excess demand anywhere in the system – which might cause money prices to return to their initial level. Hence if any set of money prices is an equilibrium set, any multiple of this set must also be an equilibrium set. The absolute price level is indeterminate." [Italics by me.]

original level. Consequently, Patinkin's real-balance-effect ensures the determinacy of the price level and, at the same time, is consistent with Walras' Law.

However, Patinkin's elegant solution of the determinacy of the price level came to fall by Weil's (1991) intertemporal analysis, which shows that in a standard Ricardian (infinitely-lived representative agent) economy, even outside money holdings cannot be net wealth. Weil's proof is analogous to Barro's (1974) proof that government bonds are no net wealth under such circumstances: If the government increases its consumption[5] and finances this by issuing government bonds, the representative household receives, on one hand, additional interest payments from these bonds plus the face value at the end of maturity. On the other hand, the present value of these payments equals exactly the additional future taxes, which the household has to pay to finance these interest payments plus redemption. Consequently the net present value of holding these bonds is zero for the household. Analogously, if the government increases its consumption and finances this by paying with banknotes, the representative household does, on one hand, *not* have to pay additional future taxes to finance any interest payments or the redemption, but receives, on the other hand, *no* interest payments and no repayment of the face value from holding these banknotes. Consequently the net present value of holding these banknotes in a Ricardian economy is for the same reasons zero as the net present value of holding government bonds.[6]

Weil (1991) shows that this zero net wealth character of money in a Ricardian economy is one explanation for the monetary superneutrality property of the Sidrauski (1967) model. He furthermore shows that this property can be overcome, if the same model is based on a non-Ricardian (overlapping generation without bequest motives) economy. Bénassy (2007) studies the effect of the zero net wealth character of money on price determinacy and shows that it is *also* a source of price *in*determinacy. Following the lines of Weil (1991), he is able to show

[5] If instead of government consumption lump sum transfers to households are assumed, the argument does not change. The only difference in this case is that the disposable income of households stays constant at the end of the day, while in the case of government consumption, the disposable income is reduced.

[6] Another (mathematically equivalent) formulation of the argument that money cannot be net wealth, is that the present value of the opportunity costs of money holdings, i.e. the forgone interest payments, equals the value of these money holdings (Weil (1991, p.37)). This formulation shows in a very clear way that a change of money prices cannot cause a real-balance-effect: An increase of money prices decreases the real value of money holdings, but also the real value of the opportunity costs of money holdings such that the net effect of an increase of money prices is zero and vice versa.

that this price indeterminacy is overcome in non-Ricardian models, because in non-Ricardian models money is net wealth such that Patinkin's real-balance-effect actually works.

Consequently, so far the basic result of economic reasoning on the problem of money price determinacy is that in order to have a wealth effect that ensures money price determinacy the assumption of a non-Ricardian economy is necessary. This means, money price determinacy depends de facto on the *non*-altruism between old and new generations. It also means that only in economies with a growing population, money price determinacy is ensured by a positive relationship between money supply and money prices, while in economies with shrinking populations the relationship between money supply and money prices becomes negative.

However, the perhaps most unsatisfying aspect of wealth-effect-based money price determinacy is its dependency on *outside money*.[7] Outside money is created, if the government buys goods in exchange for the banknotes, which it has printed, or if the government injects these bank notes as a transfer into the household budget constraints ("helicopter drops" or "Santa-Clause money"). This is, however, not the procedure by which most modern central banks bring their money into circulation.[8] Modern central banks inject most of their money in form of credits supplied to capital markets. Only a small share is injected via purchases of gold and other goods. Therefore, the largest part of money in circulation in modern economies is *inside money*. Since inside money is a credit from the central bank in cannot be net wealth to the private sector. Consequently, inside money can, even in a non-Ricardian economy, not provide the wealth-effect necessary to generate money price determinacy.

Given all the problems of finding a mechanism that ensures money price determinacy some researchers have given up the idea that money prices are well determined. So Woodford (2003, p. 34) cites Wicksell (1898, pp. 100-101), "who compares relative prices to a pendulum that always returns to the same equilibrium position when perturbed, while the money prices of goods in general are compared to a cylinder resting on a horizontal plane, which can remain equally well in any location on the plane to which it may happen to be moved".

[7] I use terms "outside" and "inside" money following Gurley and Shaw (1960).

[8] The Federal Reserve is certainly an exception here, since, at least in normal times, it injects its most of its money via the purchase of government bonds. To the contrary, the European Central bank *must not* buy "government debt instruments" nor grand "overdraft facilities" to governmental institutions by article 101 §1 of the Treaty Establishing the European Community.

However, this paper shows that money prices are always well determined in a general equilibrium framework – in the same sense relative prices are always well determined. To do so, I go back to the beginning of the debate and show in section 2 with an astonishingly simple example that – no matter whether money is introduced as inside or outside money – Walras' Law does simply not hold in monetary economies, but only in pure barter economies. This invalidity of Walras' Law in monetary economies leaves always an equation left to determine the money prices unambiguously.[9] As the discussion will show the economic intuition for this result is straightforward. Section 3 summarizes the discussion.

## 2. The Basic Argument

To focus on the basic argument, I use a simple two market endowment economy.[10] Consider first the case of a pure barter economy. The representative household receives an endowment of $Y_t$ goods, which she supplies to herself and the government via the goods market.[11] She is taxed by the government with a lump sum tax $T_t$ and has to decide how to split up her disposable income between consumption $C_t$ and savings $\Delta B_t$. Her interest income from last period savings equals $i_{t-1}$ $B_{t-1}$. The resulting budget constraint of the household equals then:

$$Y_t = C_t + T_t - i_{t-1}\, B_{t-1} + \Delta B_{D,t} \tag{1}$$

Since it is not necessary to quantify the savings decision for the following argument, an arbitrary consumption function can be chosen. The government receives no endowment with goods and consumes an amount of goods equal to $G_t$. The government budget constraint equals then:

[9] As is well known, the equality of the number of equations and the number of unknowns is only sufficient for the existence of a unique solution if the equation system is linear and the coefficient matrix of the linear equations is non-singular. I will refer to this property in the following as the "counting criterion". I will of course not discuss the problem of the existence of a general market equilibrium stemming from other sources. The reference for this discussion is Debreu (1959).

[10] As will be seen by the logic of the argument, a generalization towards a more realistic model is straightforward. Maurer (2008) provides the calculations for a DSGE model with goods market, capital market, labor market, production and capital accumulation.

[11] Of course, such a self-supply of goods via the goods market does not make much sense within the framework of an endowment economy. The endowment economy assumption is however only a simplification to clarify the basic idea. In a more realistic model, households would receive income from firms in exchange for their factor services and use this income to demand goods on the goods market and supply credits to the credit market. Firms would supply their goods to the goods market, where households, firms and the government would demand them. Maurer (2008) provides the calculations for such a more realistic economy.

$$G_t = T_t - i_{t-1}\ B_{t-1} + \Delta B_{S,t} \qquad (2)$$

The market equilibrium condition for the capital market is:

$$\Delta B_{S,t} = \Delta B_{D,t} \qquad (3)$$

If the government and the household keep their budget constraints and the capital market is in equilibrium, the goods market too must necessarily be in equilibrium. To show this, household consumption demand $C_t$ and government consumption demand $G_t$ are added up to total demand for goods. Substituting the corresponding expressions from the budget constraints for $C_t = Y_t - T_t + i_{t-1}\ B_{t-1} - \Delta B_{D,t}$ and $G_t = T_t - i_{t-1}\ B_{t-1} + \Delta B_{S,t}$, the following expression for the total demand for goods results:

$$\begin{aligned} & C_t + G_t = Y_t - T_t + i_{t-1}\ B_{t-1} - \Delta B_{D,t} + T_t - i_{t-1}\ B_{t-1} + \Delta B_{S,t} \\ & \Leftrightarrow \\ & C_t + G_t = Y_t - \Delta B_{D,t} + \Delta B_{S,t} \end{aligned} \qquad (4)$$

Using the capital market equilibrium condition (3) this shows in fact that goods demand equals goods supply:

$$C_t + G_t = Y_t \qquad (5)$$

Consequently, if the household and the government keep their budget constraints *and* there is a capital market equilibrium, the goods market too is necessarily in equilibrium. In other words, Walras' Law does hold in such a pure barter economy.

### 2.1. The Outside Money Case

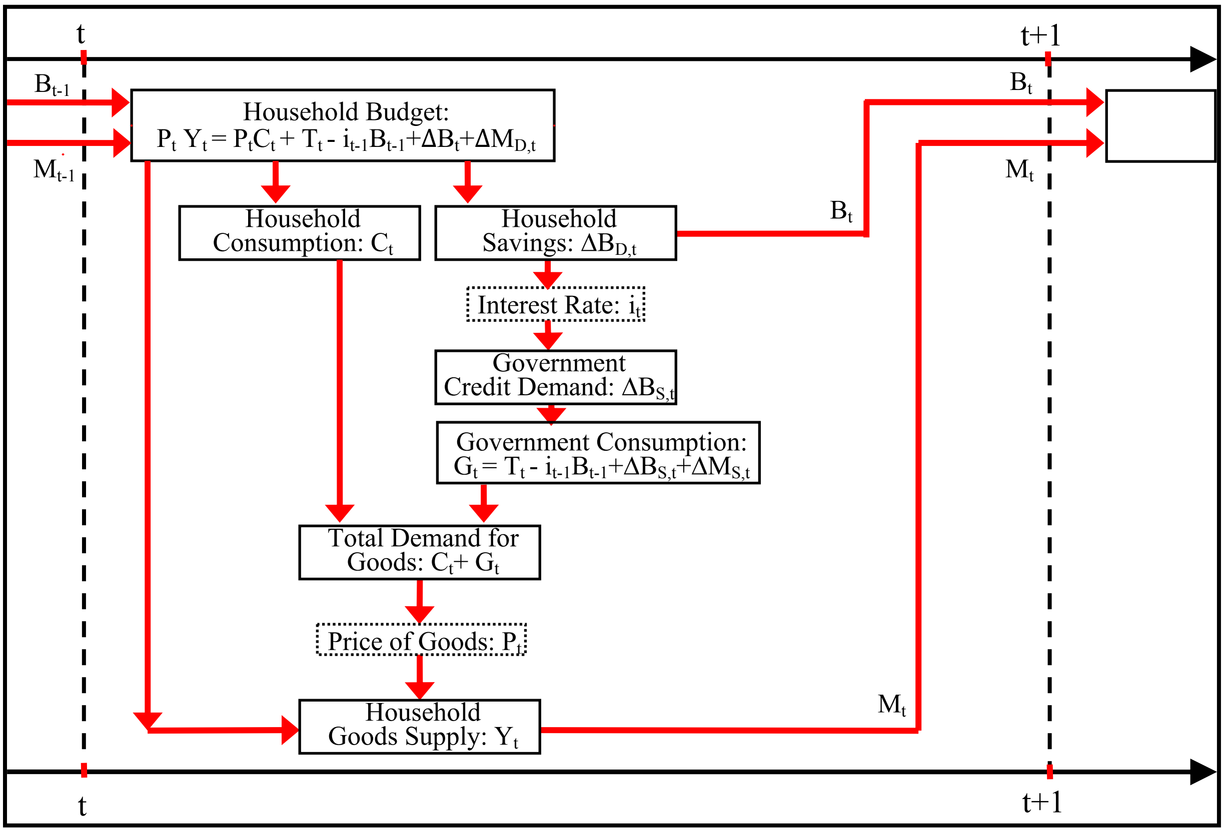
Next, consider the case, of a monetary economy, where all transactions are carried out with money as a means of exchange. Figure 1 displays the transactions in this economy. The representative household arrives in period t with her accumulated government bonds $B_{t-1}$ plus her accumulated money stock $M_{t-1}$. She receives her per period endowment with goods $Y_t$, which she supplies to the goods market. She pays her taxes $T_t$, receives the interest payments for her accumulated government bonds $i_{t-1}$ $B_{t-1}$ and decides how to split up her disposable income between consumption $C_t$, additional government bonds $\Delta B_t$ and additional money holdings $\Delta M_t$. The resulting budget constraint is then given by the following equation:

$$P_t \; Y_t = P_t \; C_t + T_t - i_{t-1} \; B_{t-1} + \Delta B_{D,t} + \Delta M_{D,t} \qquad (6)$$

The government receives no endowment with goods and consumes an amount of goods equal to $G_t$. In a monetary economy, where banknotes printed by the government are accepted as a means of payment, the government is now able to finance its consumption either by nominal taxes $T_t$, issuing additional nominal bonds $\Delta B_t$ or printing additional money $\Delta M_t$. The government budget constraint reads then:

$$P_t G_t = T_t - i_{t-1} \; B_{t-1} + \Delta B_{S,t} + \Delta M_{S,t} \qquad (7)$$

**Figure 1 - The Economic Transactions in the Outside Money Case**



If government consumption minus issuance of new bonds $P_tG_t - \Delta B_{S,t}$ is smaller than the increase of money supply $\Delta M_{S,t}$, tax payments $T_t$ will be negative such that households receive money transfers by the government. This is the popular case of "helicopter drops" or "Santa Clause money", used to engineer money into the household budged constraint in most monetary DSGE models. Whatever value $G_t$ has, money injected this way into the economy is always outside money in the sense of Gurley and Shaw (1960). In what follows, the sign of $T_t$ is of no importance for the final result. The equilibrium condition for the capital market is:

$$\Delta B_{S,t} = \Delta B_{D,t} \tag{8}$$

If the government and the household keep their budget constraints such that the consumption demand of households equals $P_t\ C_t = P_t\ Y_t - T_t + i_{t-1}\ B_{t-1} - \Delta B_{D,t} - \Delta M_{D,t}$ and $P_tG_t = T_t - i_{t-1}\ B_{t-1} + \Delta B_{S,t} + \Delta M_{S,t}$, the total demand for goods equals now:

$$\begin{aligned} & P_tC_t + P_tG_t = P_t\ Y_t - T_t + i_{t-1}\ B_{t-1} - \Delta B_{D,t} + T_t - i_{t-1}\ B_{t-1} + \Delta B_{S,t} + \Delta M_{S,t} \\ & \Leftrightarrow \\ & P_tC_t + P_tG_t = P_t\ Y_t - \Delta B_{D,t} - \Delta M_{D,t} + \Delta B_{S,t} + \Delta M_{S,t} \end{aligned} \tag{9}$$

Using the capital market equilibrium condition (8) this yields now the following equation:

$$C_t + G_t = Y_t + \frac{M_{S,t} - M_{D,t}}{P_t} \tag{10}$$

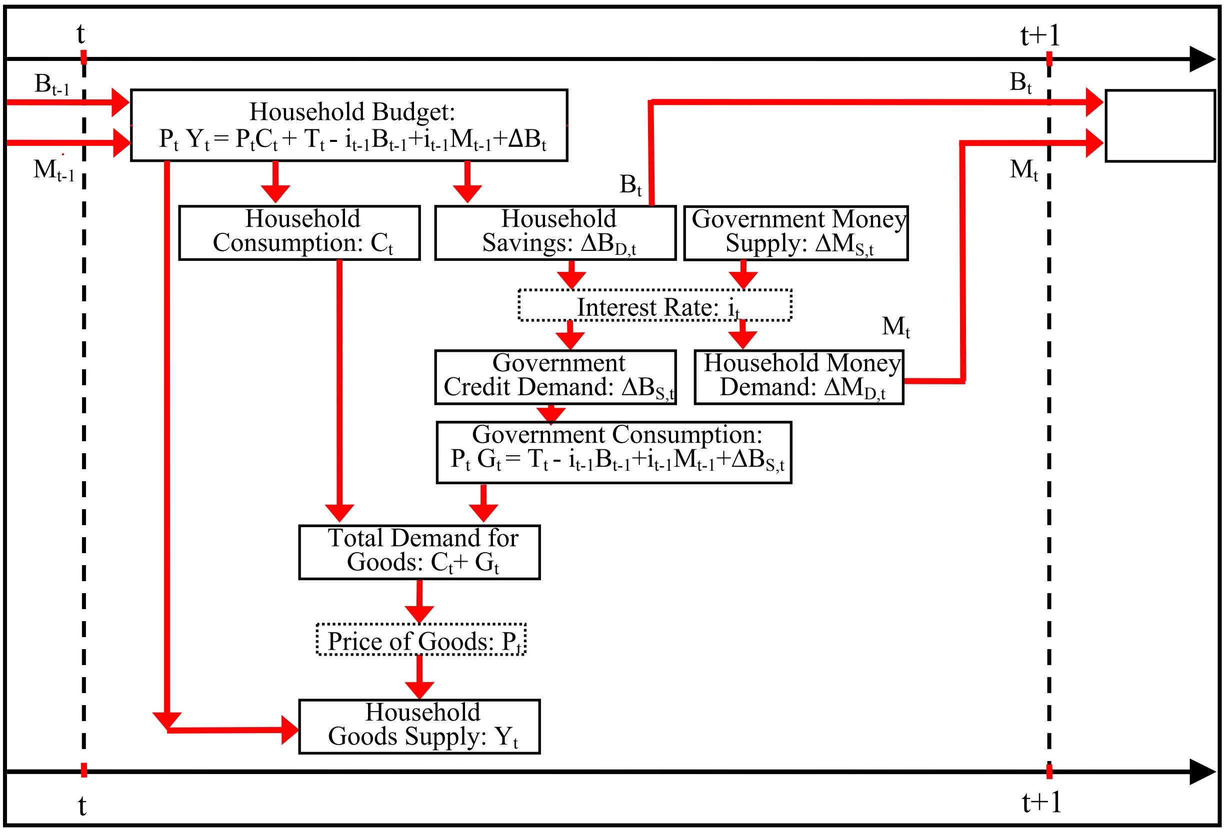
This equation shows that despite of the fact that the household and the government keep their budget constraints *and* there is a capital market equilibrium, the goods market in this monetary economy is not necessarily in equilibrium. Consequently, Walras' Law does not necessarily hold in this monetary economy. It does only hold, if money supply exactly equals money demand $M_{S,t} = M_{D,t}$. In this case the demand for goods $C_t + G_t$ will equal the supply of goods $C_t + G_t = Y_t$, as equation (10) shows. If however money supply is larger than money demand $M_{S,t} > M_{D,t}$, the demand for goods will be larger than the supply of goods

$C_t + G_t = Y_t + \frac{M_{S,t} - M_{D,t}}{P_t} > Y_t$. Before I discuss a possible adjustment process in case of such an excess supply of money, the next section shows that equation (10) also results in the inside money case.

## 2.2. The Inside Money Case

In the inside money case, money is injected into the economy as a credit supplied by the government to the credit market. Since households in a monetary economy transfer their savings with the help of money too, there is no discernable difference between a credit supplied by the government based on printing money and a credit supplied by the household based on abstained consumption. Figure 2 displays the transactions in this economy.

**Figure 2 - The Economic Transactions in the Inside Money Case**



The representative household arrives in period t with her accumulated government bonds $B_{t-1}$ plus her borrowed money stock $M_{t-1}$. The budget constraint equals then:

$$P_t\ Y_t + M_{t-1} = P_t\ C_t + T_t - i_{t-1}\ B_{t-1} + (1 + i_{t-1})\, M_{t-1} + \Delta B_{D,t} \tag{11}$$

The government consumes an amount of goods equal to $G_t$. Since the assumption is now that money is injected as a credit, the government offers the money it wants to supply to the credit market $M_{S,t}$ and receives each period an interest payment (the seignorage) from the household $i_{t-1}M_{t-1}$.

$$P_t G_t = T_t - i_{t-1}\ B_{t-1} + i_{t-1} M_{t-1} + \Delta B_{S,t} \tag{12}$$

The equilibrium condition for the capital market is now:

$$\Delta B_{D,t} + M_{S,t} = \Delta B_{S,t} + M_{D,t} \tag{13}$$

If the government and the household keep their budget constraints such that the consumption demand of households equals $P_t\ C_t = P_t\ Y_t - T_t + i_{t-1}\ B_{t-1} - i_{t-1}\ M_{t-1} - \Delta B_{D,t}$ and $P_t G_t = T_t - i_{t-1}\ B_{t-1} + i_{t-1} M_{t-1} + \Delta B_{S,t}$ , the total demand for goods equals now:

$$\begin{aligned} &P_t\ C_t + P_t G_t = P_t\ Y_t - T_t + i_{t-1}\ B_{t-1} - i_{t-1} M_{t-1} - \Delta B_{D,t} + T_t - i_{t-1}\ B_{t-1} + i_{t-1}\ M_{t-1} + \Delta B_{S,t} \\ &\Leftrightarrow \\ &P_t\ C_t + P_t G_t = P_t\ Y_t - \Delta B_{D,t} + \Delta B_{S,t} \end{aligned} \tag{14}$$

Inserting now the equilibrium condition for the capital market (13) yields once again equation (10):

$$C_t + G_t = Y_t + \frac{M_{S,t} - M_{D,t}}{P_t} \tag{10}$$

Consequently, no matter whether money is injected as outside or as inside money Walras' Law does not necessarily hold in a monetary economy. Equation (10) shows that Walras' Law only holds, if money supply does equal money demand. If money supply is larger than money demand $M_{S,t} > M_{D,t}$, demand for goods will be larger than supply of goods $C_t + G_t = Y_t + \frac{M_{S,t} - M_{D,t}}{P_t} > Y_t$, i.e. there will be excess demand for goods and vice versa. Consequently, monetary policy via inside or outside money can cause excess demand on the goods market.[12] Given the conventional assumption that the price mechanism on goods markets works sufficiently well, a positive excess demand for goods $M_{S,t} > M_{D,t}$ will cause an increase of goods prices, while a negative excess demand for goods $M_{S,t} < M_{D,t}$ will cause a decrease of goods prices. Under the standard assumption that the money demand function depends on the nominal transaction volume of the economy, the adjustment of goods prices will finally lead to a market equilibrium: If households do not only have to pay their consumption with money but also their taxes and their bond purchases, the transaction demand for money will equal the nominal value of the endowment with goods divided by the circulation velocity of money. Hence, neglecting other factors of influence, the money demand function will be:[13]

$$M_{D,t} = P_t Y_t / v_t \tag{15}$$

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[12] One interesting aspect of the inside money case is that not all credits offered at the capital market have the potential to affect the price level, but only those that are based on printed money. Credits based on household savings are "*backed*" by the production of goods. They can therefore not generate excess demand for goods. Credits based on printed money are "*not backed"* by the production of goods. They can cause excess demand for goods if their supply is larger than their demand. Maurer (2008) presents a DSGE model with demand deposit money creation by business banks depending on the cash ratio of the economy and the cash supply by the central bank. Applying the same "balance mechanics" used to derive equation (10), the model shows that demand deposit money is also not backed by the production of goods and has therefore also the potential to affect the price level. To the contrary, credits intermediated by business banks, i.e. households savings, are backed by the production of goods and can therefore not affect the price level. It is central bank cash in circulation plus demand deposit money, i.e. the monetary aggregate called M1, which is able to affect the price level, but not M2 or M3. Consequently, the balance mechanics approach applied here to derive equation (10) delivers also a theoretical criterion for the relevant monetary aggregate.

[13] The assumption of a money demand function like (15) does also imply an assumption about the timing of economic transactions. With equation (15) the standard textbook simultaneity assumption is made, i.e. all transactions of period t are paid with money, which is borrowed in period t: The household arrives at period t, borrows $M_{D,t}$ from the capital market, uses $M_{D,t}$ to buy her consumption goods $C_t$, pay her taxes $T_t$ and transfer her savings $\Delta B_t$ to the government via the capital market. An alternative, and perhaps somewhat more realistic, timing assumption would be a money demand function like $M_{D,t} = P_t \, E_t\left[\left(1 + \pi_{t+1}\right) Y_{t+1} / v_{t+1}\right]$ with $\pi_{t+1} = P_{t+1} / P_t - 1$. In this case the money used for transactions in period t must be borrowed in period t-1. However the conclusions drawn here would not change under such an assumption. See Maurer (2008) for the corresponding calculations for a DSGE model with production and capital accumulation, which is based on the latter timing assumption.

Consequently, under this money demand function the excess demand on the goods market disappears, if goods prices reach a level, where money demand equals money supply:

$$M_{S,t} = M_{D,t} = P_t Y_t / v_t \quad \Leftrightarrow \quad P_t = \frac{M_{S,t} v_t}{Y_t} \tag{16}$$

Hence the goods market is in equilibrium if the "Cambridge equation" holds. This shows that it is a "transaction volume effect" and not a "wealth effect" that restores the equilibrium between money demand and money supply.[14]

Why does Walras' Law hold in a mere barter economy but not in a monetary economy? There is a simple intuition: In a mere barter economy the government cannot simply use printed money to pay its consumption. Instead it has either to tax households or to borrow money from households. Household taxes as well as household savings are, however, "backed" by the production of goods: A household receives its income from the selling of production factors, which are used to produce goods. Consequently, government consumption demand, which is financed with household taxes or household savings, always meets the corresponding supply of goods on the goods market. Therefore, in a pure barter economy the goods market will be in equilibrium, if all the other markets of the economy are in equilibrium and everybody keeps its budget constraint. In a monetary economy with *outside money*, however, the government can finance his demand for goods by printing money. This printed money is, contrary to taxes or credits from households, not "backed" by the production of goods. Therefore it is able to produce excess demand, even if the all the other markets of the economy are in equilibrium and everybody keeps its budget constraint. In the *inside money* case the same argument holds:

[14] If the price level is well determined in a general equilibrium model, why then did Bénassy (2007) come to the conclusion that there is price level indeterminacy in Ricardian economies? The answer is that Bénassy neglected the relationship described by equation (10). He bases his indeterminacy result on the derivation of equation $D_t P_t Y_t = (1-\beta)\sum_{s=t}^{\infty} D_s P_s Y_s \ \forall t, \text{ with } D_t = \prod_{s=0}^{t-1} 1/(1+i_s)$ for the same endowment economy as described here in section 2.1 with $G_t = 0$. This formula clearly shows that if one sequence ($P_t$, $\forall t$) is a solution of this equation, then any multiple of $P_t$ will also be a solution. However in deriving this equation he assumes that a per period equilibrium on the goods market implies $C_t = Y_t$ (see Bénassy (2007, p. 6-7). As equation (10) shows, this assumption is not justified. According to equation (10), a goods market equilibrium implies the equivalence of money demand and money supply, and this condition unambiguously determines the price level in each period.

If a household supplies his savings to the capital market, these savings are "backed" by the production of goods. Consequently, if the borrower uses these household savings to demand goods, this demand is met by the supply coming from the corresponding production of goods. If, however, the government supplies credits to the capital market based on printed money, this supply of credits is not "backed" by the production of goods. Consequently, if the borrower uses these "money based" credits to demand goods, this demand is not met by a supply coming from the corresponding production of goods.[15] This shows once again, if it is possible to generate demand for goods by printing money, Walras' Law will not hold.

## 3. Conclusions

The preceding analysis has shown that in monetary economies, where money is provided by the government via buying goods and demanded by households in exchange for goods, Walras' Law does not hold: If N-1 markets are in equilibrium the keeping of all budget constraints does not necessarily imply the there will be an equilibrium on the Nth market. If N-1 markets are in equilibrium and the budget constraints are kept, the Nth market will only be in equilibrium, if money supply is equal to money demand.

The proof that Walras' Law does not hold in monetary economies is extremely simple as shown in the preceding section for an economy with two markets and in the appendix for an economy with N markets. Why has the viewpoint that Walras' Law holds in monetary economies been prevailing in economic theory so far? Most probably Walras and his followers were victims of a semantic trap. They mingled up the talking about a "money market" with the existence of a "money market", a market where money supply and money demand met and the equilibrium price of money is determined in the same way as a market for petroleum exists, where the strength of supply and demand determine the price of petroleum. But money has no own market. It slips into the economy using other markets – either goods markets as in the

[15] One may of course ask, why – following equation (10) – it is only the difference between money supply and money demand which can cause excess demand for goods and not the whole amount of money supply, since it is the whole supply of money, which is not backed by the production of goods. The answer lies in the implicit assumption that the money demanded does not create demand for goods: It is the money people hold for unforeseen transaction purposes on average in their money bags, cash boxes, vaults and the like. Therefore, even though money is used to buy goods, money demand does not create demand for goods. This idea is formalized in the inventory approach of the Baumol-Tobin model (Baumol (1952), Tobin (1956)).

outside money case or capital markets as in the inside money case. But there is no such thing as a “money market”.

## 4. Appendix

This appendix provides the same argument as given in section 2 for the case of an economy with N goods and H households. This framework is also used by Lange (1942) and Patinkin (1949) and (1965). I first show that “Walras’ Law” holds in a mere barter economy. Then I show that “Walras’ Law” does not hold in a monetary economy. Finally, I discuss the consequences.

In a barter economy households’ budgets equal:

$$\sum_{i=1}^{N} p_i \, d_{i,h} = \sum_{i=1}^{N} p_i \, s_{i,h} \quad , \qquad h = 1,...H \tag{17}$$

where $d_{i,h}$ represents the quantity of good i demanded by household h and $s_{i,h}$ represents the quantity of good i supplied by household h and $p_i$ represents the price of good i expressed in some arbitrarily chosen accounting unit. Summing up over all households yields what Lange (1942, p.50) calls “Walras’ Law”:

$$\sum_{h=1}^{H} \sum_{i=1}^{N} p_i \, d_{i,h} = \sum_{h=1}^{H} \sum_{i=1}^{N} p_i \, s_{i,h} \tag{18}$$

Rearranging yields:

$$\sum_{i=1}^{N} p_i \sum_{h=1}^{H} d_{i,h} = \sum_{i=1}^{N} p_i \sum_{h=1}^{H} s_{i,h} \tag{19}$$

$$\sum_{i=1}^{N} p_i \, D_i = \sum_{i=1}^{N} p_i \, S_i$$

where $D_i$ represents total demand of all households for good i and $S_i$ represents total supply of all households of good i. From this follows what Patinkin (1965, p.35) calls “Walras’ Law”:[16] If there are N-1 markets in equilibrium,

[16] Lange (1942, p.50, fn. 2) himself hints to the fact that Walras’ proof of what Lange calls Walras’ Law follows

$$p_i\ D_i(p_1, p_1, ... p_{N-1}) = p_i\ S_i(p_1, p_1, ... p_{N-1}) \quad \text{for all } i = 1, ... N-1$$
$$<=> \qquad (20)$$
$$\sum_{i=1}^{N-1} p_i\ D_i = \sum_{i=1}^{N-1} p_i\ S_i$$

then the N[th] market too must be in equilibrium, as a subtraction of the sum of all market equilibria from the sum of all budget constraints shows:

$$\sum_{i=1}^{N} p_i\ D_i - \sum_{i=1}^{N-1} p_i\ D_i = \sum_{i=1}^{N} p_i\ S_i - \sum_{i=1}^{N-1} p_i\ S_i$$
$$<=> \qquad (21)$$
$$p_N\ D_N = p_N\ S_N$$

Consequently, if all households keep their budgets, only N-1 independent equations exist. As is well known, if the equation system is linear and the coefficient matrix of the linear equations is non-singular, the equality of the number of equations and the number of unknowns is sufficient for the existence of a unique solution. I will refer to this property in the following as the “counting criterion”. Consequently, in a barter economy, where the counting criterion applies, it is only possible to determine N-1 prices. This implies that only the relative prices of the N goods can be determined. The relative prices can be determined by dividing the prices of all goods expressed in the accounting unit through the price of an arbitrarily chosen numéraire good. If the Nth good is the numéraire, the resulting relative prices equal $p_i/p_N$ with i = 1,2,3,…,N, where the relative price of the numéraire is normalized to equal $1 = p_N/p_N$ and has therefore not to be determined by an equation. Consequently, in a mere barter economy all prices are well determined if the counting criterion applies.

Consider now a monetary economy with a government (or central bank), where money is provided by the government via buying goods. The change of the government’s money supply is given by $\Delta M_S = M_S - M_S^0$, with $M_S^0$ equal to the stock of money supplied by the government in the past. Money is demanded by households in exchange for goods. The change of households’ money demand is given by $\Delta M_D = M_D - M_D^0 =$

the lines, which Patinkin (1965, p.35) calls Walras’ Law: Walras “has proved the theorem that if demand equals supply for n-1 commodities it does so also for the nth commodity”. See also Walras (1874, p. 184).

$\sum_{h=1}^{H} \Delta m_{D,h} = \sum_{h=1}^{H} \left( m_{D,h} - m_{D,h}^{0} \right)$, with $m_{D,h}^{0}$ equal to the stock of money accumulated by household h over the past. The budget constraint of a household equals then:

$$\sum_{i=1}^{N} p_i \, d_{i,h} + \Delta m_{D,h} = \sum_{i=1}^{N} p_i \, s_{i,h} \quad , \qquad h = 1,...H \tag{22}$$

The government budget equals:

$$\sum_{i=1}^{N} p_i \, d_{i,G} = \sum_{i=1}^{N} p_i \, s_{i,G} + \Delta M_S \tag{23}$$

Taxes are omitted since they would not influence the result. In principle, the government can use money to buy goods or bonds from households.[17] In the first case, the government provides outside money, in the second case the government provides inside money, according to the definition of Gurley and Shaw (1960). As the analysis will show, the difference between inside and outside money plays no role for the following result. Adding up all budgets yields now:

$$\begin{aligned}
&\sum_{h=1}^{H} \sum_{i=1}^{N} p_i \, d_{i,h} + \Delta m_{D,h} + \sum_{i=1}^{N} p_i \, d_{i,G} = \sum_{h=1}^{H} \sum_{i=1}^{N} p_i \, s_{i,h} + \sum_{i=1}^{N} p_i \, s_{i,G} + \Delta M_S \\
&\Leftrightarrow \\
&\sum_{h=1}^{H} \Delta m_{D,h} + \sum_{i=1}^{N} p_i \left( \sum_{h=1}^{H} d_{i,h} + d_{i,G} \right) = \sum_{i=1}^{N} \sum_{h=1}^{H} p_i \left( s_{i,h} + s_{i,G} \right) + \Delta M_S \\
&\Leftrightarrow \\
&\Delta M_D + \sum_{i=1}^{N} p_i \left( \sum_{h=1}^{H} d_{i,h} + d_{i,G} \right) = \sum_{i=1}^{N} p_i \left( \sum_{h=1}^{H} s_{i,h} + s_{i,G} \right) + \Delta M_S \\
&\Leftrightarrow \\
&\Delta M_D + \sum_{i=1}^{N} p_i D_i = \sum_{i=1}^{N} p_i S_i + \Delta M_S \\
&\Leftrightarrow \\
&\sum_{i=1}^{N} p_i D_i = \sum_{i=1}^{N} p_i S_i + \Delta M_S - \Delta M_D
\end{aligned} \tag{24}$$

Making use of the assumption that there was a monetary equilibrium in the past such that $M_S^0 = M_D^0$ equation (24) can be rewritten in the following way:

[17] Technically the purchase of bonds from a household would cause a government demand for bonds $d_{b,G}$ at a price $p_b = 1/(1+i)$. At the end of the maturity of the bond, the government would supply this bond to the household $s_{b,G}$ and receive a price of $p_b=1$.

$$\sum_{i=1}^{N} p_i D_i = \sum_{i=1}^{N} p_i S_i + M_S - M_D \tag{25}$$

Following the traditional Cambridge equation, money demand depends primarily on the nominal transaction volume, where v equals the velocity of money assumed to be equal for all households:

$$M_D = \sum_{h=1}^{H} m_{D,h} = \sum_{h=1}^{H} \frac{1}{v} \sum_{i=1}^{N} p_i \ d_{i,h} = \frac{1}{v} \sum_{i=1}^{N} p_i D_i \tag{26}$$

Inserting equation (26) in equation (25) yields finally:

$$\sum_{i=1}^{N} p_i D_i = \sum_{i=1}^{N} p_i S_i + M_S - \frac{1}{v} \sum_{i=1}^{N} p_i D_i \tag{27}$$

As this equation shows, if money supply by the government is larger than money demand by households, total demand for goods will be larger than total supply of goods by the difference between money supply and money demand $M_S - M_D$ and vice versa. Consequently, if money is supplied by the government via buying goods, there exists no "money market", where a disequilibrium results, if money supply is larger than money demand. Instead, the value of the total demand for goods will be larger than the value of the total supply of goods. In case of excess supply of money by the government, the resulting excess demand for goods will cause the money prices of goods to grow. Since money demand depends on the money prices $p_i$ of all goods, this will cause money demand to grow. When money demand equals money supply, the total demand for goods will equal the total supply of goods and the adjustment of money prices will come to a standstill. Hence the assumption of a fictitious "money market" (Lange (1942, p. 49), Patinkin (1949, p. 6)), where money demand and money supply are balanced by a fictitious "price of money" is not only counterfactual but also superfluous.

Equation (25) does also show that "Walras' Law", in the sense of Patinkin (1965), does not hold, if money is supplied by the government via buying goods: If there are N-1 markets in equilibrium,

$$
\begin{aligned}
&p_i\ D_i(p_1, p_2, \ldots p_{N-1}) = p_i\ S_i(p_1, p_2, \ldots p_{N-1}) \quad \text{for all } i = 1, \ldots N-1 \\
&<=> \\
&\sum_{i=1}^{N-1} p_i\ D_i = \sum_{i=1}^{N-1} p_i\ S_i
\end{aligned} \tag{28}
$$

then the N[th] market will not be in equilibrium, if money demand does not equal to money supply, as subtraction of the sum of all market equilibria from the sum of all budget constraints shows:

$$
\begin{aligned}
&M_D + \sum_{i=1}^{N} p_i D_i - \sum_{i=1}^{N-1} p_i\ D_i = \sum_{i=1}^{N} p_i S_i\ - \sum_{i=1}^{N-1} p_i\ S_i + M_S \\
&<=> \\
&M_D + \sum_{i=1}^{N} p_i D_i - \sum_{i=1}^{N-1} p_i\ D_i = \sum_{i=1}^{N} p_i S_i\ - \sum_{i=1}^{N-1} p_i\ S_i + M_S \\
&<=> \\
&p_N\ D_N = p_N\ S_N + M_S - M_D
\end{aligned} \tag{29}
$$

There will only be an equilibrium on the N[th] market, if money supplied by the government $M_S$ equals money demanded by households $M_D$. If the government supplies more money than is demanded by households to finance its budget, there will be excess demand on the N[th] market, even if all other markets are in equilibrium. Consequently, *at least one* market will be in disequilibrium, if money supplied by the government is larger than money demanded by households. Of course if money supply is larger than money demand, it is not very likely that the resulting excess demand for goods will hit one goods market only. It is much more likely that this excess demand will spread across more than just one market – depending on the preferences of households and government.

On whatever markets an excess supply of money will cause an excess demand for goods, it is clear that the above equation system contains N independent equations, since the keeping of the budgets by households and the government do not imply that the Nth market is in equilibrium, if N-1 markets are in equilibrium.[18] Consequently, if the counting criterion

[18] In the terminology of Lange (1942, p.52) in an economy where, for whatever reasons, money supply is always equal to money demand, is an economy where "Say's Law" does hold. As shown by Lange (1942, p.61-66), in an economy, where Say's Law holds, the number of equations n-1 is smaller by 1 than the number of unknown variables n. According to Lange, in such an economy it is therefore not possible to determine the money prices of goods: "Either Say's Law is assumed and money prices are indeterminate or money prices are made determinate – but then Say's Law precludes any monetary theory." Lange (1942, p.65-66). As the above example shows, in an economy where money is used by the government to finance its budget (at least in part), Say's Law in the sense of Lange (1942) does not hold.

applies, it will be possible to determine the N "money prices" of all goods. As equation (29) shows, if the money prices of all N goods have reached their equilibrium level such that all N goods markets are in equilibrium, money supply will necessarily equal money demand. In other words, in a monetary economy with N markets a necessary condition for a general market equilibrium is an equilibrium on all N markets. In this case money supply will also be equal to money demand. Synonymously, in a monetary economy where N-1 markets are in equilibrium *and* money supply is equal to money demand the Nth market too will be in equilibrium.

Another consequence of the injection of money by the government via buying goods or private bonds is that the classical assumption of degree 0 homogeneity of the demand and supply functions is no longer possible, since it will lead to a logical contradiction. For, even if $D_i(\lambda p_1, \lambda p_2, ... \lambda p_N) = \lambda^0\, D_i(p_1, p_2, ... p_N)$ and $S_i(\lambda p_1, \lambda p_2, ... \lambda p_N) = \lambda^0\, S_i(p_1, p_2, ... p_N)$, a multiplication of all money prices by factor λ does not leave the demand for goods unchanged, since the increase of money demand will cause a decrease of the demand for goods, if the government keeps its money supply constant. Inserting the demand and supply functions in equation (27) and assuming a general market equilibrium yields:

$$\sum_{i=1}^{N} p_i\, \left(D_i(p_1, p_2, ... p_N) - S_i(p_1, p_2, ... p_N)\right) = M_S - \frac{1}{V}\sum_{i=1}^{N} p_i D_i(p_1, p_2, ... p_N) = 0 \qquad (30)$$

Multiplying all prices by factor λ and applying the homogeneity of degree 0 assumption gives:

$$\sum_{i=1}^{N} \lambda p_i\, \left(D_i(\lambda p_1, \lambda p_2, .., \lambda p_N) - S_i(\lambda p_1, \lambda p_2, .., \lambda p_N)\right) = M_S - \frac{1}{V}\sum_{i=1}^{N-1} \lambda p_i\; D_i(\lambda p_1, p\lambda_1, .., \lambda p_N) = 0$$

$$\lambda \sum_{i=1}^{N} p_i\, \left(D_i(p_1, p_2, .., p_N) - S_i(p_1, p_2, .., p_N)\right) = M_S - \lambda \frac{1}{V}\sum_{i=1}^{N-1} p_i\; D_i(p_1, p_2, .., p_N) = 0$$

(31)

Since implies $D_i(p_1, p_2, ... p_N) = S_i(p_1, p_2, ... p_N)$ and $M_S = \frac{1}{V}\sum_{i=1}^{N} p_i D_i(p_1, p_2, ... p_N)$ equation (31) is contradictory for any $\lambda \neq 1$:

$$
\begin{aligned}
0 &= M_S - \lambda \frac{1}{v} \sum_{i=1}^{N-1} p_i \; D_i(p_1, p_2, .., p_N) \\
0 &= M_S - \lambda M_S \\
0 &= 1 - \lambda
\end{aligned}
\tag{32}
$$

Consequently, the classical dichotomy does not hold in an economy where the government injects money via buying goods or bonds. Therefore, Patinkin (1949) is right: In a monetary economy the classical dichotomy cannot hold. However, contrary to Patinkin (1949, p. 23-24) it is not necessary to introduce a “wealth effect” of “real money balances” to determine money prices. If the government (or central bank) introduces money via buying goods or private bonds and households demand for money depends on the nominal transaction volume of the economy, an excess demand for goods will result, if money supply is larger than money demand such. And it is the possibility of this excess demand that makes the money prices of goods determinate.

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